

解きながら
楽しむ

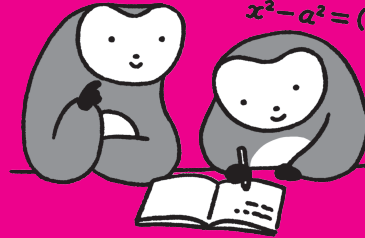
大人の
数学

$$x^2 + (a+b)x + ab = (x+a)(x+b)$$

$$x^2 + 2ax + a^2 = (x+a)^2$$

$$x^2 - 2ax + a^2 = (x-a)^2$$

$$x^2 - a^2 = (x+a)(x-a)$$



- ： 因数分解と
- ： 平方根 編

別冊解答書

答えと解き方

KUMON

1

P.4-5

 $(a+b)(c+d)$ の計算をしよう!

- 1 (1) ア、ウ
(2) イ、エ
- 2 (1) $2bd+cd$ (2) $2pr+3qr$
(3) $3cy-4dy$ (4) $-2ax+4bx$
- 3 (1) $xy+6x+3y+18$
(2) $ab+2a+8b+16$
(3) $ax+ay+bx+by$
(4) $2ac+ad+2bc+bd$
(5) $ac-ad+2bc-2bd$
(6) $5ax+5ay-bx-by$
(7) $4ax-8ay-bx+2by$
(8) $-3am-9bm+an+3bn$
(9) $20ac-35ad+12bc-21bd$
(10) $-5ac+3ad+10bc-6bd$

解き方

- 1 $2a^2$ や $-2xyz$ のように、数や文字を掛け合わせた式を単項式、 $4a-b$ や x^2-5y^2-1 のように、単項式の足し算の形で表された式を多項式といいます。
- 2 文字と文字の掛け算は、ふつうアルファベット順に書きます。
- (1) $d(2b+c)=d \times 2b+d \times c$
 $=2bd+cd$
- (2) $(2p+3q)r=2p \times r+3q \times r$
 $=2pr+3qr$
- (4) $-2x(a-2b)=-2x \times a-2x \times (-2b)$
 $=-2ax+4bx$
- 3 (1) $(x+3)(y+6)=x(y+6)+3(y+6)$
 $=xy+6x+3y+18$
- (3) $(a+b)(x+y)=a(x+y)+b(x+y)$
 $=ax+ay+bx+by$
- (5) $(a+2b)(c-d)=a(c-d)+2b(c-d)$
 $=ac-ad+2bc-2bd$
- (6) $(5a-b)(x+y)=5a(x+y)-b(x+y)$
 $=5ax+5ay-bx-by$
- (7) $(4a-b)(x-2y)=4a(x-2y)-b(x-2y)$
 $=4ax-8ay-bx+2by$
- (10) $(-a+2b)(5c-3d)$
 $=-a(5c-3d)+2b(5c-3d)$
 $=-5ac+3ad+10bc-6bd$

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P.6-7

 $(a+b)(x+y+z)$ の計算をしよう!

- 1 (1) $2ax+ay+az+2bx+by+bz$
(2) $2ax+2ay-2az+bx+by-bz$
(3) $ac-2ad+3ae-3bc+6bd-9be$
(4) $6ax-9ay+15az+8bx-12by+20bz$
(5) $2ax+6x+ay+3y+a+3$
(6) $3ax-bx-6ay+2by+9a-3b$
- 2 (1) $A+B \cdots 6x^2+3x+2$
 $A-B \cdots 4x^2+x$
(2) $A+B \cdots -x^3+6x^2+9x-10$
 $A-B \cdots -5x^3+8x^2-x+4$

解き方

- 1 (1) $(a+b)(2x+y+z)$
 $=a(2x+y+z)+b(2x+y+z)$
 $=2ax+ay+az+2bx+by+bz$
- (3) $(a-3b)(c-2d+3e)$
 $=a(c-2d+3e)-3b(c-2d+3e)$
 $=ac-2ad+3ae-3bc+6bd-9be$
- (5) $(2x+y+1)(a+3)$
 $=2x(a+3)+y(a+3)+(a+3)$
 $=2ax+6x+ay+3y+a+3$
- 2 (1) $A+B=(5x^2+2x+1)+(x^2+x+1)$
 $=(5+1)x^2+(2+1)x+(1+1)$
 $=6x^2+3x+2$
 $A-B=(5x^2+2x+1)-(x^2+x+1)$
 $=(5-1)x^2+(2-1)x+(1-1)$
 $=4x^2+x$
- (2) $A+B=(-3x^3+7x^2+4x-3)$
 $\quad\quad\quad+(2x^3-x^2+5x-7)$
 $=(-3+2)x^3+(7-1)x^2$
 $\quad\quad\quad+(4+5)x+(-3-7)$
 $=-x^3+6x^2+9x-10$
 $A-B=(-3x^3+7x^2+4x-3)$
 $\quad\quad\quad-(2x^3-x^2+5x-7)$
 $=(-3-2)x^3+(7+1)x^2$
 $\quad\quad\quad+(4-5)x+(-3+7)$
 $=-5x^3+8x^2-x+4$

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P.8-9

指数法則って何？

- 1 (1) a^5 (2) a^{20}
 (3) a^{11} (4) a^{24}
 (5) a^8 (6) a^{14}
- 2 (1) $-12x^6$ (2) $-64x^6$
 (3) $-18a^8$ (4) $-40a^6$
 (5) $-14x^2y^3$ (6) $27x^6y^{12}$
 (7) $2x^7y^3$ (8) $81x^5y^5$

解き方

- 1 (1) $a^2 \times a^3 = (a \times a) \times (a \times a \times a) = a^{2+3} = a^5$
 (2) $(a^4)^5 = (a \times a \times a \times a)^5 = a^{4 \times 5} = a^{20}$
 (3) $(a^2)^3 \times a^5 = a^{2 \times 3} \times a^5 = a^6 \times a^5 = a^{6+5} = a^{11}$
 (4) $\{(a^3)^2\}^4 = (a^{3 \times 2})^4 = (a^6)^4 = a^{6 \times 4} = a^{24}$
 (5) $a^2 \times a \times a^5 = a^{2+1+5} = a^8$
 (6) $a \times (a^2)^2 \times (a^3)^3 = a \times a^{2 \times 2} \times a^{3 \times 3}$
 $= a \times a^4 \times a^9$
 $= a^{1+4+9} = a^{14}$
- 2 (1) $-3x^2 \times 4x^4 = -3 \times 4 \times x^{2+4} = -12x^6$
 (2) $(-4x^2)^3 = (-4)^3 \times x^{2 \times 3} = -64x^6$
 (3) $(-3a^3)^2 \times (-2a^2) = 9a^6 \times (-2a^2)$
 $= -18a^8$
 (4) $a^2 \times 5a \times (-2a)^3 = a^2 \times 5a \times (-8a^3)$
 $= -40a^6$
 (5) $-7x^2y \times 2y^2 = (-7) \times 2 \times x^2 \times y^{1+2}$
 $= -14x^2y^3$
 (6) $(3x^2y^4)^3 = 3^3 \times x^{2 \times 3} \times y^{4 \times 3} = 27x^6y^{12}$
 (7) $2x^3y \times x^4y^2 = 2 \times x^{3+4} \times y^{1+2} = 2x^7y^3$
 (8) $(3x^2y)^2 \times 9xy^3 = 9x^4y^2 \times 9xy^3 = 81x^5y^5$

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P.10-11

分配法則と指数法則を使って式を展開しよう！

- 1 (1) $-6x^3 - 12x^2 + 24x$
 (2) $x^4 - 4x^3 + 5x^2$
 (3) $x^2y^4 - 8xy^3$
 (4) $2x^4y - 7x^2y^2$
 (5) $6x^5 + 13x^4 + 5x^3$
 (6) $-8x^5 + 30x^4 - 7x^3$
- 2 (1) $x^3 - x^2 - x - 2$
 (2) $x^3 + x^2 - 2x + 12$
 (3) $2x^3 - 15x^2 + 12x - 35$
 (4) $2x^3 + x^2 - 9$
 (5) $a^4 + a^3 + 2a^2 + a + 1$
 (6) $a^3 + 2a^2b + 2ab^2 + b^3$
 (7) $3x^3 - 10x^2y + 6xy^2 - y^3$
 (8) $x^4 + x^3y + 2x^2y^2 + xy^3 + y^4$

解き方

- 1 (5) $(2x^3 + x^2)(3x^2 + 5x)$
 $= 2x^3(3x^2 + 5x) + x^2(3x^2 + 5x)$
 $= 6x^5 + 10x^4 + 3x^4 + 5x^3$
 $= 6x^5 + 13x^4 + 5x^3$
- (6) $(7x^2 - 2x^3)(4x^2 - x)$
 $= 7x^2(4x^2 - x) - 2x^3(4x^2 - x)$
 $= 28x^4 - 7x^3 - 8x^5 + 2x^4$
 $= -8x^5 + 30x^4 - 7x^3$
- 2 (1) $(x-2)(x^2+x+1)$
 $= x(x^2+x+1) - 2(x^2+x+1)$
 $= x^3+x^2+x-2x^2-2x-2$
 $= x^3-x^2-x-2$
- (3) $(2x^2-x+5)(x-7)$
 $= 2x^2(x-7) - x(x-7) + 5(x-7)$
 $= 2x^3 - 14x^2 - x^2 + 7x + 5x - 35$
 $= 2x^3 - 15x^2 + 12x - 35$
- (4) $(x^2+2x+3)(2x-3)$
 $= x^2(2x-3) + 2x(2x-3) + 3(2x-3)$
 $= 2x^3 - 3x^2 + 4x^2 - 6x + 6x - 9$
 $= 2x^3 + x^2 - 9$
- (6) $(a+b)(a^2+ab+b^2)$
 $= a(a^2+ab+b^2) + b(a^2+ab+b^2)$
 $= a^3 + a^2b + ab^2 + a^2b + ab^2 + b^3$
 $= a^3 + 2a^2b + 2ab^2 + b^3$

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P.12-13

確認問題①

- 1 (1) $3ac+4ad+6bc+8bd$
 (2) $6ac-18ad-2bc+6bd$
 (3) $12ax+6bx+3cx+20ay+10by+5cy$
 (4) $7ax-21bx-7cx-6ay+18by+6cy$
- 2 (1) $A+B\cdots x^2+2x+3$
 $A-B\cdots 3x^2-8x+5$
 (2) $A+B\cdots -7x^2-6x-8$
 $A-B\cdots 2x^3-9x^2-4x+4$
- 3 (1) $-5a^3b^3$ (2) $-8x^8y^9$
 (3) $-108a^7$ (4) $16x^{12}y^8$
- 4 (1) x^3+9x^2+7x-8
 (2) $a^5-a^4+4a^3-a^2+a-4$
 (3) $a^3-2a^2b-5ab^2+6b^3$
 (4) $8x^4-10x^3y+3x^2y^2-12xy^3+9y^4$

解き方

- 1 (3) $(3x+5y)(4a+2b+c)$
 $=3x(4a+2b+c)+5y(4a+2b+c)$
 $=12ax+6bx+3cx+20ay+10by+5cy$
- 2 (2) $A+B=(x^3-8x^2-5x-2)$
 $+(-x^3+x^2-x-6)$
 $= (1-1)x^3+(-8+1)x^2$
 $+(-5-1)x+(-2-6)$
 $= -7x^2-6x-8$
 $A-B=(x^3-8x^2-5x-2)$
 $-(-x^3+x^2-x-6)$
 $= (1+1)x^3+(-8-1)x^2$
 $+(-5+1)x+(-2+6)$
 $= 2x^3-9x^2-4x+4$
- 3 (3) $2a^4 \times (-6a) \times (-3a)^2$
 $= 2a^4 \times (-6a) \times (-3)^2 \times a^2$
 $= 2 \times (-6) \times 9 \times a^{4+1+2}$
 $= -108a^7$
- 4 (4) $(2x^3-x^2y-3y^3)(4x-3y)$
 $= 2x^3(4x-3y)-x^2y(4x-3y)$
 $\quad -3y^3(4x-3y)$
 $= 8x^4-6x^3y-4x^3y+3x^2y^2-12xy^3+9y^4$
 $= 8x^4-10x^3y+3x^2y^2-12xy^3+9y^4$

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P.14-15

 $(a+b)^2, (a-b)^2$ を展開してみよう!①

- 1 (1) a^2+2a+1 (2) $a^2+16a+64$
 (3) a^2-4a+4 (4) $a^2-10a+25$
 (5) $4x^2+4x+1$ (6) $9x^2-6x+1$
 (7) $9a^2+12a+4$ (8) $25x^2-30x+9$
- 2 (1) $4a^2+4ab+b^2$ (2) $25x^2-10xy+y^2$
 (3) $a^2+12ab+36b^2$ (4) $x^2-14xy+49y^2$
 (5) $25a^2-20ab+4b^2$ (6) $4x^2+12xy+9y^2$
 (7) $9a^4+6a^2b+b^2$ (8) $x^2-10xy^2+25y^4$
 (9) $16a^4+24a^2b^2+9b^4$ (10) $49x^4-42x^2y^2+9y^4$

解き方

- 1 (1) $(a+1)^2=a^2+2\cdot a\cdot 1+1^2=a^2+2a+1$
 (2) $(a+8)^2=a^2+2\cdot a\cdot 8+8^2=a^2+16a+64$
 (3) $(a-2)^2=a^2-2\cdot a\cdot 2+2^2=a^2-4a+4$
 (4) $(a-5)^2=a^2-2\cdot a\cdot 5+5^2=a^2-10a+25$
 (5) $(2x+1)^2=(2x)^2+2\cdot (2x)\cdot 1+1^2$
 $=4x^2+4x+1$
 (6) $(3x-1)^2=(3x)^2-2\cdot (3x)\cdot 1+1^2$
 $=9x^2-6x+1$
 (7) $(3a+2)^2=(3a)^2+2\cdot (3a)\cdot 2+2^2$
 $=9a^2+12a+4$
 (8) $(5x-3)^2=(5x)^2-2\cdot (5x)\cdot 3+3^2$
 $=25x^2-30x+9$
- 2 (1) $(2a+b)^2=(2a)^2+2\cdot (2a)\cdot b+b^2$
 $=4a^2+4ab+b^2$
 (2) $(5x-y)^2=(5x)^2-2\cdot (5x)\cdot y+y^2$
 $=25x^2-10xy+y^2$
 (3) $(a+6b)^2=a^2+2\cdot a\cdot (6b)+(6b)^2$
 $=a^2+12ab+36b^2$
 (4) $(x-7y)^2=x^2-2\cdot x\cdot (7y)+(7y)^2$
 $=x^2-14xy+49y^2$
 (5) $(5a-2b)^2=(5a)^2-2\cdot (5a)\cdot (2b)+(2b)^2$
 $=25a^2-20ab+4b^2$
 (6) $(2x+3y)^2=(2x)^2+2\cdot (2x)\cdot (3y)+(3y)^2$
 $=4x^2+12xy+9y^2$
 (7) $(3a^2+b)^2=(3a^2)^2+2\cdot (3a^2)\cdot b+b^2$
 $=9a^4+6a^2b+b^2$
 (9) $(4a^2+3b^2)^2=(4a^2)^2+2\cdot (4a^2)\cdot (3b^2)+(3b^2)^2$
 $=16a^4+24a^2b^2+9b^4$
 (10) $(7x^2-3y^2)^2=(7x^2)^2-2\cdot (7x^2)\cdot (3y^2)+(3y^2)^2$
 $=49x^4-42x^2y^2+9y^4$

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P.16-17

 $(a+b)^2, (a-b)^2$ を展開してみよう!②

- 1 (1) $a^2+2ab+b^2+10a+10b+25$
 (2) $a^2-2ab+b^2-4a+4b+4$
 (3) $4x^2+4xy+y^2+4x+2y+1$
 (4) $x^2-4xy+4y^2-6x+12y+9$
 (5) $9a^2-12ab+4b^2+36a-24b+36$
 (6) $4x^2+16xy+16y^2-20x-40y+25$
- 2 (1) $x^2+2xy+y^2+2xz+2yz+z^2$
 (2) $a^2+2ab+b^2-6ac-6bc+9c^2$
 (3) $x^2-6xy+9y^2-10xz+30yz+25z^2$
 (4) $x^6+2x^5-3x^4-4x^3+4x^2$

解き方

- 1 (1) $a+b=A$ とおくと、
 $(a+b+5)^2$
 $= (A+5)^2 = A^2 + 10A + 25$
 $= (a+b)^2 + 10(a+b) + 25$
 $= a^2 + 2ab + b^2 + 10a + 10b + 25$
- (3) $2x+y=A$ とおくと、
 $(2x+y+1)^2$
 $= (A+1)^2 = A^2 + 2A + 1$
 $= (2x+y)^2 + 2(2x+y) + 1$
 $= 4x^2 + 4xy + y^2 + 4x + 2y + 1$
- (5) $3a-2b=A$ とおくと、
 $(3a-2b+6)^2$
 $= (A+6)^2 = A^2 + 12A + 36$
 $= (3a-2b)^2 + 12(3a-2b) + 36$
 $= 9a^2 - 12ab + 4b^2 + 36a - 24b + 36$
- 2 (1) $(x+y+z)^2 = \{(x+y)+z\}^2$
 $= (x+y)^2 + 2(x+y)z + z^2$
 $= x^2 + 2xy + y^2 + 2xz + 2yz + z^2$
- (2) $(a+b-3c)^2 = \{(a+b)-3c\}^2$
 $= (a+b)^2 - 2(a+b) \cdot (3c) + (3c)^2$
 $= a^2 + 2ab + b^2 - 6ac - 6bc + 9c^2$
- (3) $(x-3y-5z)^2 = \{(x-3y)-5z\}^2$
 $= (x-3y)^2 + 2(x-3y) \cdot (-5z) + (5z)^2$
 $= x^2 - 6xy + 9y^2 - 10xz + 30yz + 25z^2$
- (4) $(x^3+x^2-2x)^2 = \{(x^3+x^2)-2x\}^2$
 $= (x^3+x^2)^2 - 2(x^3+x^2) \cdot (2x) + 4x^2$
 $= x^6 + 2x^5 + x^4 - 4x^4 - 4x^3 + 4x^2$
 $= x^6 + 2x^5 - 3x^4 - 4x^3 + 4x^2$

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P.18-19

 $(a+b)(a-b), (x+a)(x+b)$ を展開してみよう!①

- 1 (1) a^2-9 (2) x^2-25
 (3) $64-a^2$ (4) x^2-y^2
 (5) $9x^2-4$ (6) a^4-4a^2
 (7) x^2-25y^2 (8) $49a^2-4b^2$
- 2 (1) $x^2+10x+21$ (2) $a^2+9a+20$
 (3) $a^2+4a-12$ (4) $x^2+5x-14$
 (5) x^2-7x-8 (6) $a^2+6a-27$
 (7) $a^2-11a+28$ (8) $x^2-11x+30$
 (9) $2a^2-14a-36$ (10) $-3x^2+15x-18$

解き方

- 1 (1) $(a+3)(a-3) = a^2 - 3^2 = a^2 - 9$
 (2) $(x-5)(x+5) = x^2 - 5^2 = x^2 - 25$
 (3) $(8-a)(8+a) = 8^2 - a^2 = 64 - a^2$
 (5) $(3x-2)(3x+2) = (3x)^2 - 2^2 = 9x^2 - 4$
 (6) $(a^2+2a)(a^2-2a) = (a^2)^2 - (2a)^2 = a^4 - 4a^2$
 (7) $(x+5y)(x-5y) = x^2 - (5y)^2 = x^2 - 25y^2$
 (8) $(7a+2b)(7a-2b) = (7a)^2 - (2b)^2 = 49a^2 - 4b^2$
- 2 (1) $(x+3)(x+7) = x^2 + (3+7)x + 3 \cdot 7$
 $= x^2 + 10x + 21$
- (2) $(a+4)(a+5) = a^2 + (4+5)a + 4 \cdot 5$
 $= a^2 + 9a + 20$
- (3) $(a+6)(a-2) = a^2 + (6-2)a + 6 \cdot (-2)$
 $= a^2 + 4a - 12$
- (4) $(x+7)(x-2) = x^2 + (7-2)x + 7 \cdot (-2)$
 $= x^2 + 5x - 14$
- (5) $(x-8)(x+1) = x^2 + (-8+1)x + (-8) \cdot 1$
 $= x^2 - 7x - 8$
- (6) $(a-3)(a+9) = a^2 + (-3+9)a + (-3) \cdot 9$
 $= a^2 + 6a - 27$
- (7) $(a-4)(a-7) = a^2 + (-4-7)a + (-4) \cdot (-7)$
 $= a^2 - 11a + 28$
- (8) $(x-5)(x-6) = x^2 + (-5-6)x + (-5) \cdot (-6)$
 $= x^2 - 11x + 30$
- (9) $2(a+2)(a-9) = 2\{a^2 + (2-9)a + 2 \cdot (-9)\}$
 $= 2(a^2 - 7a - 18)$
 $= 2a^2 - 14a - 36$
- (10) $-3(x-2)(x-3)$
 $= -3\{x^2 + (-2-3)x + (-2) \cdot (-3)\}$
 $= -3(x^2 - 5x + 6)$
 $= -3x^2 + 15x - 18$

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P.20-21

 $(a+b)(a-b)$ 、 $(x+a)(x+b)$ を展開してみよう!②

- 1 (1) $x^2 - 2xy + y^2 - 9$
 (2) $a^2 + 2ab + b^2 - 36$
 (3) $a^2 + 4ab + 4b^2 - 25$
 (4) $9x^2 - 6xy + y^2 - 16$
 (5) $4x^2 + 12xy + 9y^2 - 1$
 (6) $16a^2 + 8ab + b^2 - c^2$
- 2 (1) $a^2 + 2ab + b^2 - 3a - 3b - 10$
 (2) $9a^2 + 6ab + b^2 - 6a - 2b - 24$
 (3) $a^2 + 8a + 16 - ab - 4b - 6b^2$
 (4) $x^4 + 4x^3 + 12x^2 + 16x - 9$
 (5) $x^4 + 5x^2y^2 + y^4 + 4x^3y + 4xy^3$

解き方

- 1 (1) $x - y = A$ とおくと、
 $(x - y - 3)(x - y + 3) = (A - 3)(A + 3)$
 $= A^2 - 3^2$
 $= (x - y)^2 - 9$
 $= x^2 - 2xy + y^2 - 9$
- (3) $a + 2b = A$ とおくと、
 $(a + 2b + 5)(a + 2b - 5) = (A + 5)(A - 5)$
 $= A^2 - 5^2$
 $= (a + 2b)^2 - 25$
 $= a^2 + 4ab + 4b^2 - 25$
- 2 (1) $a + b = A$ とおくと、
 $(a + b - 5)(a + b + 2) = (A - 5)(A + 2)$
 $= A^2 - 3A - 10$
 $= (a + b)^2 - 3(a + b) - 10$
 $= a^2 + 2ab + b^2 - 3a - 3b - 10$
- (3) $a + 4 = A$ とおくと、
 $(a + 2b + 4)(a - 3b + 4) = (A + 2b)(A - 3b)$
 $= A^2 - bA - 6b^2$
 $= (a + 4)^2 - b(a + 4) - 6b^2$
 $= a^2 + 8a + 16 - ab - 4b - 6b^2$
- (5) $x^2 + y^2 = A$ とおくと、
 $(x^2 + xy + y^2)(x^2 + 3xy + y^2)$
 $= (A + xy)(A + 3xy)$
 $= A^2 + 4xyA + 3x^2y^2$
 $= (x^2 + y^2)^2 + 4xy(x^2 + y^2) + 3x^2y^2$
 $= x^4 + 2x^2y^2 + y^4 + 4x^3y + 4xy^3 + 3x^2y^2$
 $= x^4 + 5x^2y^2 + y^4 + 4x^3y + 4xy^3$

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P.22-23

 $(ax+b)(cx+d)$ を展開してみよう!

- 1 (1) $2x^2 + 7x + 3$ (2) $2a^2 - a - 15$
 (3) $6x^2 - 11x + 4$ (4) $12a^2 + 5a - 2$
 (5) $6x^2 + 17x - 14$ (6) $10a^2 + 3a - 18$
- 2 (1) $2x^2 + 9xy + 10y^2$ (2) $3a^2 - 13ab - 10b^2$
 (3) $6x^2 + 47xy + 35y^2$ (4) $6x^2 - 11xy - 10y^2$
 (5) $20a^2 - 23ab - 21b^2$
 (6) $18a^2 + 19ab - 12b^2$
 (7) $14a^2 - 31ab + 15b^2$
 (8) $12x^4 - 7x^2y^2 - 12y^4$

解き方

- 1 (3) $(2x - 1)(3x - 4)$
 $= 2 \cdot 3x^2 + \{2 \cdot (-4) + (-1) \cdot 3\}x + (-1) \cdot (-4)$
 $= 6x^2 - 11x + 4$
- (5) $(2x + 7)(3x - 2)$
 $= 2 \cdot 3x^2 + \{2 \cdot (-2) + 7 \cdot 3\}x + 7 \cdot (-2)$
 $= 6x^2 + 17x - 14$
- (6) $(5a - 6)(2a + 3)$
 $= 5 \cdot 2a^2 + \{5 \cdot 3 + (-6) \cdot 2\}a + (-6) \cdot 3$
 $= 10a^2 + 3a - 18$
- 2 (1) $(2x + 5y)(x + 2y)$
 $= 2 \cdot 1x^2 + \{2 \cdot (2y) + (5y) \cdot 1\}x + (5y) \cdot (2y)$
 $= 2x^2 + 9xy + 10y^2$
- (2) $(3a + 2b)(a - 5b)$
 $= 3 \cdot 1a^2 + \{3 \cdot (-5b) + (2b) \cdot 1\}a + (2b) \cdot (-5b)$
 $= 3a^2 - 13ab - 10b^2$
- (4) $(2x - 5y)(3x + 2y)$
 $= 2 \cdot 3x^2 + \{2 \cdot (2y) + (-5y) \cdot 3\}x + (-5y) \cdot (2y)$
 $= 6x^2 - 11xy - 10y^2$
- (5) $(5a + 3b)(4a - 7b)$
 $= 5 \cdot 4a^2 + \{5 \cdot (-7b) + (3b) \cdot 4\}a + (3b) \cdot (-7b)$
 $= 20a^2 - 23ab - 21b^2$
- (7) $(7a - 5b)(2a - 3b)$
 $= 7 \cdot 2a^2 + \{7 \cdot (-3b) + (-5b) \cdot 2\}a$
 $\quad \quad \quad + (-5b) \cdot (-3b)$
 $= 14a^2 - 31ab + 15b^2$
- (8) $(4x^2 + 3y^2)(3x^2 - 4y^2)$
 $= 4 \cdot 3x^4 + \{4 \cdot (-4y^2) + (3y^2) \cdot 3\}x^2$
 $\quad \quad \quad + (3y^2) \cdot (-4y^2)$
 $= 12x^4 - 7x^2y^2 - 12y^4$

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P.24-25

展開の公式を利用して計算しよう!

- 1 (1) 1024 (2) 2401
 (3) 9801 (4) 65.61
- 2 27394756
- 3 (1) 9999 (2) 4899
 (3) 9975 (4) 8.96
- 4 16

解き方

- 1 (1) $32^2 = (30+2)^2 = 30^2 + 2 \cdot 30 \cdot 2 + 2^2$
 $= 900 + 120 + 4 = 1024$
- (2) $49^2 = (50-1)^2 = 50^2 - 2 \cdot 50 \cdot 1 + 1^2$
 $= 2500 - 100 + 1 = 2401$
- (3) $99^2 = (100-1)^2 = 100^2 - 2 \cdot 100 \cdot 1 + 1^2$
 $= 10000 - 200 + 1 = 9801$
- (4) $8.1^2 = (8+0.1)^2 = 8^2 + 2 \cdot 8 \cdot 0.1 + 0.1^2$
 $= 64 + 1.6 + 0.01 = 65.61$
- 2 $5234^2 = (5000+234)^2$
 $= 5000^2 + 2 \cdot 5000 \cdot 234 + 234^2$
 $= 25000000 + 2340000 + 54756$
 $= 27394756$
- 3 (1) $101 \times 99 = (100+1)(100-1) = 100^2 - 1^2$
 $= 10000 - 1 = 9999$
- (2) $71 \times 69 = (70+1)(70-1) = 70^2 - 1^2$
 $= 4900 - 1 = 4899$
- (3) $105 \times 95 = (100+5)(100-5) = 100^2 - 5^2$
 $= 10000 - 25 = 9975$
- (4) $3.2 \times 2.8 = (3+0.2)(3-0.2) = 3^2 - 0.2^2$
 $= 9 - 0.04 = 8.96$
- 4 $664^2 - 660 \times 668$
 $= 664^2 - (664-4)(664+4)$
 $= 664^2 - (664^2 - 4^2)$
 $= 664^2 - 664^2 + 4^2 = 16$

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P.26-27

確認問題②

- 1 (1) $x^2 - 4xy + 4y^2$ (2) $9a^2 + 24ab + 16b^2$
 (3) $4x^2 - 12xy + 9y^2$ (4) $a^2 - 36$
 (5) $25x^2 - 4$ (6) $4a^2 - 9b^2$
- 2 (1) $a^2 + 2ab + b^2 + 4ac + 4bc + 4c^2$
 (2) $4x^2 - 4xy + y^2 - 4xz + 2yz + z^2$
 (3) $x^2 + 4xy + 4y^2 - x - 2y - 6$
 (4) $x^4 - 6x^3 + 3x^2 + 18x - 7$
- 3 (1) $6a^2 + 13ab - 5b^2$
 (2) $18x^2 + 41xy - 10y^2$
 (3) $6a^2b^2 - 5abc - 6c^2$
 (4) $15a^2 - 19ab + 6b^2$
- 4 (1) 841 (2) 2499

解き方

- 1 (2) $(3a+4b)^2 = (3a)^2 + 2 \cdot (3a) \cdot (4b) + (4b)^2$
 $= 9a^2 + 24ab + 16b^2$
- (6) $(2a-3b)(2a+3b) = (2a)^2 - (3b)^2$
 $= 4a^2 - 9b^2$
- 2 (1) $(a+b+2c)^2 = \{(a+b)+2c\}^2$
 $= (a+b)^2 + 2 \cdot (a+b) \cdot (2c) + (2c)^2$
 $= a^2 + 2ab + b^2 + 4ac + 4bc + 4c^2$
- (4) $x^2 - 3x = A$ とおくと、
 $(x^2 - 3x - 7)(x^2 - 3x + 1) = (A - 7)(A + 1)$
 $= A^2 - 6A - 7$
 $= (x^2 - 3x)^2 - 6(x^2 - 3x) - 7$
 $= x^4 - 6x^3 + 9x^2 - 6x^2 + 18x - 7$
 $= x^4 - 6x^3 + 3x^2 + 18x - 7$
- 3 (1) $(2a+5b)(3a-b)$
 $= 2 \cdot 3a^2 + \{2 \cdot (-b) + (5b) \cdot 3\}a$
 $\qquad\qquad\qquad + (5b) \cdot (-b)$
 $= 6a^2 + 13ab - 5b^2$
- (3) $(3ab+2c)(2ab-3c)$
 $= 3 \cdot 2a^2b^2 + \{3 \cdot (-3c) + (2c) \cdot 2\}ab$
 $\qquad\qquad\qquad + (2c) \cdot (-3c)$
 $= 6a^2b^2 - 5abc - 6c^2$
- 4 (1) $29^2 = (30-1)^2 = 30^2 - 2 \cdot 30 \cdot 1 + 1^2$
 $= 900 - 60 + 1 = 841$
- (2) $51 \times 49 = (50+1)(50-1) = 50^2 - 1^2$
 $= 2500 - 1 = 2499$

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P.28-29

 $(a+b)^3$ 、 $(a-b)^3$ を展開してみよう!①

- 1 (1) $a^3+6a^2+12a+8$
 (2) $x^3+12x^2+48x+64$
 (3) a^3-3a^2+3a-1
 (4) $x^3-6x^2+12x-8$
 (5) $a^3+15a^2+75a+125$
 (6) $x^3-9x^2+27x-27$

- 2 (1) $a^3+6a^2b+12ab^2+8b^3$
 (2) $a^3+9a^2b+27ab^2+27b^3$
 (3) $x^3-3x^2y+3xy^2-y^3$
 (4) $x^3-6x^2y+12xy^2-8y^3$
 (5) $a^3+18a^2b+108ab^2+216b^3$
 (6) $x^3-21x^2y+147xy^2-343y^3$
 (7) $x^3-24x^2y+192xy^2-512y^3$
 (8) $a^3-12a^2b+48ab^2-64b^3$

解き方

$$1 (1) (a+2)^3 = a^3 + 3 \cdot a^2 \cdot 2 + 3 \cdot a \cdot 2^2 + 2^3 \\ = a^3 + 6a^2 + 12a + 8$$

$$(3) (a-1)^3 = a^3 - 3 \cdot a^2 \cdot 1 + 3 \cdot a \cdot 1^2 - 1^3 \\ = a^3 - 3a^2 + 3a - 1$$

$$2 (1) (a+2b)^3 = a^3 + 3 \cdot a^2 \cdot (2b) + 3 \cdot a \cdot (2b)^2 + (2b)^3 \\ = a^3 + 6a^2b + 12ab^2 + 8b^3$$

$$(2) (a+3b)^3 = a^3 + 3 \cdot a^2 \cdot (3b) + 3 \cdot a \cdot (3b)^2 + (3b)^3 \\ = a^3 + 9a^2b + 27ab^2 + 27b^3$$

$$(3) (x-y)^3 = x^3 - 3 \cdot x^2 \cdot y + 3 \cdot x \cdot y^2 - y^3 \\ = x^3 - 3x^2y + 3xy^2 - y^3$$

$$(4) (x-2y)^3 \\ = x^3 - 3 \cdot x^2 \cdot (2y) + 3 \cdot x \cdot (2y)^2 - (2y)^3 \\ = x^3 - 6x^2y + 12xy^2 - 8y^3$$

$$(5) (a+6b)^3 \\ = a^3 + 3 \cdot a^2 \cdot (6b) + 3 \cdot a \cdot (6b)^2 + (6b)^3 \\ = a^3 + 18a^2b + 108ab^2 + 216b^3$$

$$(6) (x-7y)^3 \\ = x^3 - 3 \cdot x^2 \cdot (7y) + 3 \cdot x \cdot (7y)^2 - (7y)^3 \\ = x^3 - 21x^2y + 147xy^2 - 343y^3$$

$$(7) (x-8y)^3 \\ = x^3 - 3 \cdot x^2 \cdot (8y) + 3 \cdot x \cdot (8y)^2 - (8y)^3 \\ = x^3 - 24x^2y + 192xy^2 - 512y^3$$

$$(8) (a-4b)^3 \\ = a^3 - 3 \cdot a^2 \cdot (4b) + 3 \cdot a \cdot (4b)^2 - (4b)^3 \\ = a^3 - 12a^2b + 48ab^2 - 64b^3$$

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P.30-31

 $(a+b)^3$ 、 $(a-b)^3$ を展開してみよう!②

- 1 (1) $8a^3+12a^2+6a+1$
 (2) $125x^3+150x^2+60x+8$
 (3) $27a^3-54a^2+36a-8$
 (4) $27x^3-27x^2+9x-1$
 (5) $8a^3-84a^2+294a-343$
 (6) $64x^3+144x^2+108x+27$
 (7) $x^6+6x^4+12x^2+8$
 (8) $-a^3-3a^2-3a-1$

- 2 (1) $8a^3+12a^2b+6ab^2+b^3$
 (2) $27x^3-27x^2y+9xy^2-y^3$
 (3) $8a^3+36a^2b+54ab^2+27b^3$
 (4) $125a^3-225a^2b+135ab^2-27b^3$
 (5) $-a^6+9a^4-27a^2+27$
 (6) $-8a^3-12a^2b-6ab^2-b^3$

解き方

$$1 (2) (5x+2)^3 \\ = (5x)^3 + 3 \cdot (5x)^2 \cdot 2 + 3 \cdot (5x) \cdot 2^2 + 2^3 \\ = 125x^3 + 150x^2 + 60x + 8$$

$$(5) (2a-7)^3 \\ = (2a)^3 - 3 \cdot (2a)^2 \cdot 7 + 3 \cdot (2a) \cdot 7^2 - 7^3 \\ = 8a^3 - 84a^2 + 294a - 343$$

$$(7) (x^2+2)^3 \\ = (x^2)^3 + 3 \cdot (x^2)^2 \cdot 2 + 3 \cdot x^2 \cdot 2^2 + 2^3 \\ = x^6 + 6x^4 + 12x^2 + 8$$

$$(8) (-a-1)^3 \\ = (-a)^3 - 3 \cdot (-a)^2 \cdot 1 + 3 \cdot (-a) \cdot 1^2 - 1^3 \\ = -a^3 - 3a^2 - 3a - 1$$

$$2 (1) (2a+b)^3 \\ = (2a)^3 + 3 \cdot (2a)^2 \cdot b + 3 \cdot (2a) \cdot b^2 + b^3 \\ = 8a^3 + 12a^2b + 6ab^2 + b^3$$

$$(4) (5a-3b)^3 \\ = (5a)^3 - 3 \cdot (5a)^2 \cdot (3b) \\ \quad + 3 \cdot (5a) \cdot (3b)^2 - (3b)^3 \\ = 125a^3 - 225a^2b + 135ab^2 - 27b^3$$

$$(5) (-a^2+3)^3 \\ = (-a^2)^3 + 3 \cdot (-a^2)^2 \cdot 3 + 3 \cdot (-a^2) \cdot 3^2 + 3^3 \\ = -a^6 + 9a^4 - 27a^2 + 27$$

$$(6) (-2a-b)^3 \\ = (-2a)^3 - 3 \cdot (-2a)^2 \cdot b + 3 \cdot (-2a) \cdot b^2 - b^3 \\ = -8a^3 - 12a^2b - 6ab^2 - b^3$$

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P.32-33

 $(a+b)(a^2-ab+b^2), (a-b)(a^2+ab+b^2)$
 を展開してみよう! ①

- 1 (1) $a^6 - 12a^4 + 48a^2 - 64$
 (2) $x^6 - 27x^4 + 243x^2 - 729$
 (3) $64x^6 - 48x^4 + 12x^2 - 1$
 (4) $a^6 - 12a^4b^2 + 48a^2b^4 - 64b^6$
- 2 (1) $a^3 + 1$ (2) $x^3 + 64$
 (3) $x^3 - 27$ (4) $a^3 - 8$
 (5) $a^3 + 125$ (6) $x^3 - 216$

解き方

- 1 (1) $(a+2)^3(a-2)^3 = \{(a+2)(a-2)\}^3$
 $= (a^2-4)^3$
 $= (a^2)^3 - 3 \cdot (a^2)^2 \cdot 4 + 3 \cdot a^2 \cdot 4^2 - 4^3$
 $= a^6 - 12a^4 + 48a^2 - 64$
- (2) $(x+3)^3(x-3)^3 = \{(x+3)(x-3)\}^3$
 $= (x^2-9)^3$
 $= (x^2)^3 - 3 \cdot (x^2)^2 \cdot 9 + 3 \cdot x^2 \cdot 9^2 - 9^3$
 $= x^6 - 27x^4 + 243x^2 - 729$
- (3) $(2x+1)^3(2x-1)^3 = \{(2x+1)(2x-1)\}^3$
 $= (4x^2-1)^3$
 $= (4x^2)^3 - 3 \cdot (4x^2)^2 \cdot 1 + 3 \cdot (4x^2) \cdot 1^2 - 1^3$
 $= 64x^6 - 48x^4 + 12x^2 - 1$
- (4) $(a+2b)^3(a-2b)^3 = \{(a+2b)(a-2b)\}^3$
 $= (a^2-4b^2)^3$
 $= (a^2)^3 - 3 \cdot (a^2)^2 \cdot (4b^2)$
 $\quad + 3 \cdot a^2 \cdot (4b^2)^2 - (4b^2)^3$
 $= a^6 - 12a^4b^2 + 48a^2b^4 - 64b^6$
- 2 (1) $(a+1)(a^2-a+1) = a^3 + 1^3 = a^3 + 1$
 (2) $(x+4)(x^2-4x+16) = x^3 + 4^3 = x^3 + 64$
 (3) $(x-3)(x^2+3x+9) = x^3 - 3^3 = x^3 - 27$
 (4) $(a-2)(a^2+2a+4) = a^3 - 2^3 = a^3 - 8$
 (5) $(a+5)(a^2-5a+25) = a^3 + 5^3 = a^3 + 125$
 (6) $(x-6)(x^2+6x+36) = x^3 - 6^3 = x^3 - 216$

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P.34-35

 $(a+b)(a^2-ab+b^2), (a-b)(a^2+ab+b^2)$
 を展開してみよう! ②

- 1 (1) $a^3 + 27b^3$ (2) $x^3 + 343y^3$
 (3) $a^3 - 64b^3$ (4) $x^3 - 125y^3$
 (5) $a^3 - 1000b^3$ (6) $x^3 + 512y^3$
 (7) $a^3 - \frac{1}{27}$ (8) $x^3 + \frac{8}{27}$
- 2 (1) $8a^3 - 1$ (2) $27x^3 + 1$
 (3) $27a^3 + 8$ (4) $8x^3 - 27$
 (5) $125a^3 - 27$ (6) $64x^3 + 125$
 (7) $27a^3 + b^3$ (8) $125x^3 - y^3$
 (9) $8a^3 - 125b^3$ (10) $27x^3 + 8y^3$

解き方

- 1 (1) $(a+3b)(a^2-3ab+9b^2) = a^3 + (3b)^3$
 $= a^3 + 27b^3$
- (2) $(x+7y)(x^2-7xy+49y^2) = x^3 + (7y)^3$
 $= x^3 + 343y^3$
- (3) $(a-4b)(a^2+4ab+16b^2) = a^3 - (4b)^3$
 $= a^3 - 64b^3$
- (7) $(a-\frac{1}{3})(a^2+\frac{1}{3}a+\frac{1}{9}) = a^3 - (\frac{1}{3})^3$
 $= a^3 - \frac{1}{27}$
- (8) $(x+\frac{2}{3})(x^2-\frac{2}{3}x+\frac{4}{9}) = x^3 + (\frac{2}{3})^3$
 $= x^3 + \frac{8}{27}$
- 2 (1) $(2a-1)(4a^2+2a+1) = (2a)^3 - 1^3$
 $= 8a^3 - 1$
- (2) $(3x+1)(9x^2-3x+1) = (3x)^3 + 1^3$
 $= 27x^3 + 1$
- (3) $(3a+2)(9a^2-6a+4) = (3a)^3 + 2^3$
 $= 27a^3 + 8$
- (4) $(2x-3)(4x^2+6x+9) = (2x)^3 - 3^3$
 $= 8x^3 - 27$
- (7) $(3a+b)(9a^2-3ab+b^2) = (3a)^3 + b^3$
 $= 27a^3 + b^3$
- (8) $(5x-y)(25x^2+5xy+y^2) = (5x)^3 - y^3$
 $= 125x^3 - y^3$
- (9) $(2a-5b)(4a^2+10ab+25b^2)$
 $= (2a)^3 - (5b)^3 = 8a^3 - 125b^3$
- (10) $(3x+2y)(9x^2-6xy+4y^2)$
 $= (3x)^3 + (2y)^3 = 27x^3 + 8y^3$

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P.36-37

工夫して展開しよう!

- 1 (1) $a^2+b^2+2ab+2a+2b+1$
 (2) $a^2+4b^2+4ab-2a-4b+1$
 (3) $x^2+y^2+4z^2+2xy+4yz+4zx$
 (4) $4x^2+y^2+z^2+4xy-2yz-4zx$
 (5) $9a^2+4b^2+c^2+12ab+4bc+6ca$
 (6) $4a^2+25b^2+c^2-20ab+10bc-4ca$
- 2 (1) $x^4-2x^2y^2+y^4$
 (2) $a^4+2a^3b-3a^2b^2-4ab^3+4b^4$
 (3) $x^4+10x^3+35x^2+50x+24$
 (4) $a^4+2a^3-13a^2-14a+24$

解き方

- 1 (2) $(a+2b-1)^2$
 $=a^2+(2b)^2+(-1)^2+2\cdot a\cdot(2b)$
 $\quad +2\cdot(2b)\cdot(-1)+2\cdot(-1)\cdot a$
 $=a^2+4b^2+4ab-2a-4b+1$
- (3) $(x+y+2z)^2$
 $=x^2+y^2+(2z)^2+2\cdot x\cdot y+2\cdot y\cdot(2z)$
 $\quad +2\cdot(2z)\cdot x$
 $=x^2+y^2+4z^2+2xy+4yz+4zx$
- (5) $(3a+2b+c)^2$
 $= (3a)^2+(2b)^2+c^2+2\cdot(3a)\cdot(2b)$
 $\quad +2\cdot(2b)\cdot c+2\cdot c\cdot(3a)$
 $=9a^2+4b^2+c^2+12ab+4bc+6ca$
- 2 (1) $(x+y)^2(x-y)^2$
 $=\{(x+y)(x-y)\}^2$
 $= (x^2-y^2)^2$
 $= (x^2)^2-2\cdot x^2\cdot y^2+(y^2)^2$
 $= x^4-2x^2y^2+y^4$
- (2) $(a-b)^2(a+2b)^2$
 $=\{(a-b)(a+2b)\}^2$
 $= (a^2+ab-2b^2)^2$
 $= (a^2)^2+(ab)^2+(-2b^2)^2+2\cdot a^2\cdot(ab)$
 $\quad +2\cdot(ab)\cdot(-2b^2)+2\cdot(-2b^2)\cdot a^2$
 $= a^4+2a^3b-3a^2b^2-4ab^3+4b^4$
- (4) $(a-1)(a-3)(a+2)(a+4)$
 $= (a-1)(a+2)(a-3)(a+4)$
 $= (a^2+a-2)(a^2+a-12)$
 $= (a^2+a)^2-14(a^2+a)+24$
 $= a^4+2a^3+a^2-14a^2-14a+24$
 $= a^4+2a^3-13a^2-14a+24$

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P.38-39

確認問題③

- 1 (1) a^3+3a^2+3a+1
 (2) $x^3-12x^2+48x-64$
 (3) $x^3+6x^2y+12xy^2+8y^3$
 (4) $125a^3-150a^2b+60ab^2-8b^3$
 (5) x^3+125 (6) $27a^3-b^3$
 (7) $64x^3-27y^3$ (8) $27x^3y^3+8z^3$
- 2 (1) $a^6-3a^4b^2+3a^2b^4-b^6$
 (2) $4a^2+b^2+c^2+4ab+2bc+4ca$
 (3) $x^4+2x^3-3x^2-4x+4$
 (4) $x^4-2x^3-25x^2+26x+120$

解き方

- 1 (3) $(x+2y)^3=x^3+3\cdot x^2\cdot(2y)$
 $\quad +3\cdot x\cdot(2y)^2+(2y)^3$
 $=x^3+6x^2y+12xy^2+8y^3$
- (4) $(5a-2b)^3$
 $= (5a)^3-3\cdot(5a)^2\cdot(2b)$
 $\quad +3\cdot(5a)\cdot(2b)^2-(2b)^3$
 $= 125a^3-150a^2b+60ab^2-8b^3$
- (5) $(x+5)(x^2-5x+25)=x^3+5^3=x^3+125$
- (6) $(3a-b)(9a^2+3ab+b^2)$
 $= (3a)^3-b^3=27a^3-b^3$
- (7) $(4x-3y)(16x^2+12xy+9y^2)$
 $= (4x)^3-(3y)^3=64x^3-27y^3$
- (8) $(3xy+2z)(9x^2y^2-6xyz+4z^2)$
 $= (3xy)^3+(2z)^3=27x^3y^3+8z^3$
- 2 (1) $(a+b)^3(a-b)^3$
 $=\{(a+b)(a-b)\}^3$
 $= (a^2-b^2)^3$
 $= (a^2)^3-3\cdot(a^2)^2\cdot b^2+3\cdot a^2\cdot(b^2)^2-(b^2)^3$
 $= a^6-3a^4b^2+3a^2b^4-b^6$
- (2) $(2a+b+c)^2$
 $= (2a)^2+b^2+c^2+2\cdot(2a)\cdot b+2\cdot b\cdot c$
 $\quad +2\cdot c\cdot(2a)$
 $= 4a^2+b^2+c^2+4ab+2bc+4ca$
- (4) $(x-5)(x-3)(x+2)(x+4)$
 $= (x-5)(x+4)(x-3)(x+2)$
 $= (x^2-x-20)(x^2-x-6)$
 $= (x^2-x)^2-26(x^2-x)+120$
 $= x^4-2x^3+x^2-26x^2+26x+120$
 $= x^4-2x^3-25x^2+26x+120$

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P.40-41

共通因数をくり出そう!①

- 1 (1) $x(x+4)$ (2) $y(4x+7)$
 (3) $2a(a-3)$ (4) $-x^2(2x+5y)$
 (5) $3ab(b-6)$ (6) $5xz(5y+3)$
- 2 (1) $x(2x^2+x+1)$ (2) $a(a+b+c)$
 (3) $3a(4x-y-3z)$ (4) $ax(ax^2+3x+1)$
 (5) $xy(3x-6+y)$ (6) $xy(xy^2+y-1)$
 (7) $6axy(a^2x-6x^2y+3ay^2)$

解き方

- 1 (1) $x^2+4x=x \cdot x+x \cdot 4=x(x+4)$
 (2) $4xy+7y=y \cdot 4x+y \cdot 7=y(4x+7)$
 (3) $2a^2-6a=2a \cdot a+2a \cdot (-3)=2a(a-3)$
 (4) $-2x^3-5x^2y=-x^2 \cdot 2x-x^2 \cdot 5y$
 $=-x^2(2x+5y)$
 (5) $3ab^2-18ab=3ab \cdot b+3ab \cdot (-6)$
 $=3ab(b-6)$
 (6) $25xyz+15xz=5xz \cdot 5y+5xz \cdot 3$
 $=5xz(5y+3)$
- 2 (1) $2x^3+x^2+x=x \cdot 2x^2+x \cdot x+x \cdot 1$
 $=x(2x^2+x+1)$
 (2) $a^2+ab+ac=a \cdot a+a \cdot b+a \cdot c$
 $=a(a+b+c)$
 (3) $12ax-3ay-9az$
 $=3a \cdot 4x+3a \cdot (-y)+3a \cdot (-3z)$
 $=3a(4x-y-3z)$
 (4) $a^2x^3+3ax^2+ax$
 $=ax \cdot ax^2+ax \cdot 3x+ax \cdot 1$
 $=ax(ax^2+3x+1)$
 (5) $3x^2y-6xy+xy^2$
 $=xy \cdot 3x+xy \cdot (-6)+xy \cdot y$
 $=xy(3x-6+y)$
 (6) $x^2y^3+xy^2-xy$
 $=xy \cdot xy^2+xy \cdot y+xy \cdot (-1)$
 $=xy(xy^2+y-1)$
 (7) $6a^3x^2y-36ax^3y^2+18a^2xy^3$
 $=6axy \cdot a^2x+6axy \cdot (-6x^2y)$
 $+6axy \cdot 3ay^2$
 $=6axy(a^2x-6x^2y+3ay^2)$

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P.42-43

共通因数をくり出そう!②

- 1 (1) $(3-x)(x+1)$ (2) $(2a+b)(x+2y)$
 (3) $2(a-2b)(x-2y)$ (4) $(a-b)(a-b+2)$
 (5) $(a+b)(3ac+3bc-2)$
 (6) $(c-1)(a-b)$ (7) $2(a+2)(x-y)$
 (8) $2(2a-1)(p-2q)$
- 2 (1) $(a+b)(x+1)$ (2) $(a+1)(b-c)$
 (3) $(3a-4b)(x-y)$ (4) $(x-2)(y-z)$
 (5) $(z-x)(xy-1)$ (6) $(ab-c)(a^2-b)$

解き方

- 1 (2) $(2a+b)x+(4a+2b)y$
 $= (2a+b)x+(2a+b) \cdot 2y$
 $= (2a+b)(x+2y)$
 (3) $2a(x-2y)+4b(2y-x)$
 $= 2a(x-2y)-4b(-2y+x)$
 $= 2a(x-2y)-4b(x-2y)$
 $= (2a-4b)(x-2y)=2(a-2b)(x-2y)$
 (5) $3(a+b)^2c-2a-2b=3(a+b)^2c-2(a+b)$
 $= (a+b)\{3(a+b)c-2\}$
 $= (a+b)(3ac+3bc-2)$
 (7) $2a(x-y)+4x-4y=2a(x-y)+4(x-y)$
 $= (2a+4)(x-y)=2(a+2)(x-y)$
 (8) $4a(p-2q)-2p+4q$
 $= 4a(p-2q)-2(p-2q)$
 $= (4a-2)(p-2q)$
 $= 2(2a-1)(p-2q)$
- 2 (1) $ax+bx+a+b=(a+b)x+(a+b)$
 $= (a+b)(x+1)$
 (2) $ab+b-ac-c=ab-ac+b-c$
 $= a(b-c)+(b-c)$
 $= (a+1)(b-c)$
 (3) $3ax-4bx+4by-3ay$
 $= 3ax-3ay-4bx+4by$
 $= 3a(x-y)-4b(x-y)$
 $= (3a-4b)(x-y)$
 (5) $xyz-x^2y+x-z=xyz-z-x^2y+x$
 $= z(xy-1)-x(xy-1)$
 $= (z-x)(xy-1)$
 (6) $a^3b-a^2c+bc-ab^2=a^3b-ab^2-a^2c+bc$
 $= ab(a^2-b)-c(a^2-b)$
 $= (ab-c)(a^2-b)$

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P.44-45

 $(a+b)^2$ 、 $(a-b)^2$ を使って因数分解しよう!①

1 (1) $(a+3)^2$ (2) $(x+6)^2$

(3) $(a-2)^2$ (4) $(x-4)^2$

(5) $(a-\frac{1}{4})^2$ (6) $(x-\frac{3}{2})^2$

2 (1) $(2a+3)^2$ (2) $(4x+7)^2$

(3) $(5a-1)^2$ (4) $(3x-5)^2$

(5) $(3a-2b)^2$ (6) $(4x+\frac{1}{2}y)^2$

(7) $(2a-\frac{1}{5}b)^2$ (8) $(3xy-\frac{1}{3})^2$

解き方

1 (1) $a^2+6a+9=a^2+2\cdot 3\cdot a+3^2$
 $= (a+3)^2$

(3) $a^2-4a+4=a^2-2\cdot 2\cdot a+2^2$
 $= (a-2)^2$

(5) $a^2-\frac{1}{2}a+\frac{1}{16}=a^2-2\cdot \frac{1}{4}\cdot a+(\frac{1}{4})^2$
 $= (a-\frac{1}{4})^2$

(6) $x^2-3x+\frac{9}{4}=x^2-2\cdot \frac{3}{2}\cdot x+(\frac{3}{2})^2$
 $= (x-\frac{3}{2})^2$

2 (1) $4a^2+12a+9=(2a)^2+2\cdot (2a)\cdot 3+3^2$
 $= (2a+3)^2$

(3) $25a^2-10a+1=(5a)^2-2\cdot (5a)\cdot 1+1^2$
 $= (5a-1)^2$

(5) $9a^2-12ab+4b^2$
 $= (3a)^2-2\cdot (3a)\cdot (2b)+(2b)^2$
 $= (3a-2b)^2$

(6) $16x^2+4xy+\frac{1}{4}y^2$
 $= (4x)^2+2\cdot (4x)\cdot (\frac{1}{2}y)+(\frac{1}{2}y)^2$
 $= (4x+\frac{1}{2}y)^2$

(8) $9x^2y^2-2xy+\frac{1}{9}$
 $= (3xy)^2-2\cdot (3xy)\cdot \frac{1}{3}+(\frac{1}{3})^2$
 $= (3xy-\frac{1}{3})^2$

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P.46-47

 $(a+b)^2$ 、 $(a-b)^2$ を使って因数分解しよう!②

1 (1) $2(a-3)^2$ (2) $3(x+2)^2$

(3) $-4(x-1)^2$ (4) $-3(a-3)^2$

(5) $x(x+5)^2$ (6) $ab(a-9)^2$

2 (1) $2(2x+5y)^2$ (2) $2a(2a-3)^2$

(3) $y^2(2x+3)^2$ (4) $3ab(b-3c)^2$

(5) $3(x+y-1)^2$ (6) $a(m+n+2)^2$

(7) $(a-b)(x+3)^2$ (8) $(x+2y)(x-5)^2$

解き方

1 (1) $2a^2-12a+18=2(a^2-6a+9)=2(a-3)^2$

(2) $3x^2+12x+12=3(x^2+4x+4)=3(x+2)^2$

(3) $-4x^2+8x-4=-4(x^2-2x+1)$
 $= -4(x-1)^2$

(4) $-3a^2+18a-27=-3(a^2-6a+9)$
 $= -3(a-3)^2$

(5) $x^3+10x^2+25x=x(x^2+10x+25)$
 $= x(x+5)^2$

(6) $a^3b-18a^2b+81ab=ab(a^2-18a+81)$
 $= ab(a-9)^2$

2 (1) $8x^2+40xy+50y^2$
 $= 2(4x^2+20xy+25y^2)$
 $= 2(2x+5y)^2$

(2) $8a^3-24a^2+18a=2a(4a^2-12a+9)$
 $= 2a(2a-3)^2$

(3) $4x^2y^2+12xy^2+9y^2=y^2(4x^2+12x+9)$
 $= y^2(2x+3)^2$

(4) $3ab^3-18ab^2c+27abc^2$
 $= 3ab(b^2-6bc+9c^2)$
 $= 3ab(b-3c)^2$

(5) $3(x+y)^2-6(x+y)+3$
 $= 3\{(x+y)^2-2(x+y)+1\}=3(x+y-1)^2$

(6) $a(m+n)^2+4a(m+n)+4a$
 $= a\{(m+n)^2+4(m+n)+4\}$
 $= a(m+n+2)^2$

(7) $(a-b)x^2+6(a-b)x+9(a-b)$
 $= (a-b)(x^2+6x+9)=(a-b)(x+3)^2$

(8) $(x+2y)x^2-10(x+2y)x+25(x+2y)$
 $= (x+2y)(x^2-10x+25)$
 $= (x+2y)(x-5)^2$

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P.48-49

 $(a+b)(a-b)$ を使って因数分解しよう!①

- 1 (1) $(a+3)(a-3)$ (2) $(x+12)(x-12)$
 (3) $(2a+5)(2a-5)$ (4) $(6x+7)(6x-7)$
 (5) $\left(\frac{x}{2}+1\right)\left(\frac{x}{2}-1\right)$
 (6) $\left(\frac{a}{5}+\frac{1}{3}\right)\left(\frac{a}{5}-\frac{1}{3}\right)$
- 2 (1) $(x+2y)(x-2y)$
 (2) $(2x+7y)(2x-7y)$
 (3) $(x+yz)(x-yz)$
 (4) $(xy+4a)(xy-4a)$
 (5) $3(x+5y)(x-5y)$
 (6) $a(x+3y)(x-3y)$
 (7) $2z(y+2xz)(y-2xz)$
 (8) $3ab(2a+b)(2a-b)$

解き方

- 1 (1) $a^2 - q = a^2 - 3^2 = (a+3)(a-3)$
 (2) $x^2 - 144 = x^2 - 12^2 = (x+12)(x-12)$
 (3) $4a^2 - 25 = (2a)^2 - 5^2 = (2a+5)(2a-5)$
 (4) $36x^2 - 49 = (6x)^2 - 7^2 = (6x+7)(6x-7)$
 (5) $\frac{x^2}{4} - 1 = \left(\frac{x}{2}\right)^2 - 1^2 = \left(\frac{x}{2}+1\right)\left(\frac{x}{2}-1\right)$
 (6) $\frac{a^2}{25} - \frac{1}{9} = \left(\frac{a}{5}\right)^2 - \left(\frac{1}{3}\right)^2 = \left(\frac{a}{5}+\frac{1}{3}\right)\left(\frac{a}{5}-\frac{1}{3}\right)$
- 2 (1) $x^2 - 4y^2 = x^2 - (2y)^2 = (x+2y)(x-2y)$
 (2) $4x^2 - 49y^2 = (2x)^2 - (7y)^2 = (2x+7y)(2x-7y)$
 (3) $x^2 - y^2z^2 = x^2 - (yz)^2 = (x+yz)(x-yz)$
 (4) $x^2y^2 - 16a^2 = (xy)^2 - (4a)^2 = (xy+4a)(xy-4a)$
 (5) $3x^2 - 75y^2 = 3(x^2 - 25y^2) = 3\{x^2 - (5y)^2\} = 3(x+5y)(x-5y)$
 (6) $ax^2 - 9ay^2 = a(x^2 - 9y^2) = a\{x^2 - (3y)^2\} = a(x+3y)(x-3y)$
 (7) $2y^2z - 8x^2z^2 = 2z(y^2 - 4x^2z^2) = 2z\{y^2 - (2xz)^2\} = 2z(y+2xz)(y-2xz)$
 (8) $12a^3b - 3ab^3 = 3ab(4a^2 - b^2) = 3ab\{(2a)^2 - b^2\} = 3ab(2a+b)(2a-b)$

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P.50-51

 $(a+b)(a-b)$ を使って因数分解しよう!②

- 1 (1) $(2x^2+y)(2x^2-y)$
 (2) $(2x+3y^2)(2x-3y^2)$
 (3) $(3x^2y+4z)(3x^2y-4z)$
 (4) $-(7x^2+ab)(7x^2-ab)$
 (5) $3(a^2+3)(a^2-3)$
 (6) $4(x^2+2y^2)(x^2-2y^2)$
- 2 (1) $(a^2+1)(a+1)(a-1)$
 (2) $(x^2+9)(x+3)(x-3)$
 (3) $(a^2+4b^2)(a+2b)(a-2b)$
 (4) $2(a^4+b^2)(a^2+b)(a^2-b)$
 (5) $y^2(x^2+y^2z^2)(x+yz)(x-yz)$
 (6) $(x^4+y^4)(x^2+y^2)(x+y)(x-y)$
 (7) $(16a^4+1)(4a^2+1)(2a+1)(2a-1)$

解き方

- 1 (1) $4x^4 - y^2 = (2x^2)^2 - y^2 = (2x^2+y)(2x^2-y)$
 (2) $4x^2 - 9y^4 = (2x)^2 - (3y^2)^2 = (2x+3y^2)(2x-3y^2)$
 (3) $9x^4y^2 - 16z^2 = (3x^2y)^2 - (4z)^2 = (3x^2y+4z)(3x^2y-4z)$
 (4) $-49x^4 + a^2b^2 = -(49x^4 - a^2b^2) = -\{(7x^2)^2 - (ab)^2\} = -(7x^2+ab)(7x^2-ab)$
 (5) $3a^4 - 27 = 3(a^4 - 9) = 3\{a^2 - 3\} = 3(a^2+3)(a^2-3)$
 (6) $4x^4 - 16y^4 = 4(x^4 - 4y^4) = 4\{(x^2)^2 - (2y^2)^2\} = 4(x^2+2y^2)(x^2-2y^2)$
- 2 (1) $a^4 - 1 = (a^2)^2 - (1)^2 = (a^2+1)(a^2-1) = (a^2+1)(a+1)(a-1)$
 (2) $x^4 - 81 = (x^2)^2 - (3^2)^2 = (x^2+3^2)(x^2-3^2) = (x^2+9)(x+3)(x-3)$
 (3) $a^4 - 16b^4 = (a^2)^2 - (4b^2)^2 = (a^2+4b^2)(a^2-4b^2) = (a^2+4b^2)\{a^2 - (2b)^2\} = (a^2+4b^2)(a+2b)(a-2b)$
 (5) $x^4y^2 - y^6z^4 = y^2(x^4 - y^4z^4) = y^2\{(x^2)^2 - (y^2z^2)^2\} = y^2(x^2+y^2z^2)(x^2-y^2z^2) = y^2(x^2+y^2z^2)\{x^2 - (yz)^2\} = y^2(x^2+y^2z^2)(x+yz)(x-yz)$
 (6) $x^8 - y^8 = (x^4)^2 - (y^4)^2 = (x^4+y^4)(x^4-y^4) = (x^4+y^4)\{(x^2)^2 - (y^2)^2\} = (x^4+y^4)(x^2+y^2)(x^2-y^2) = (x^4+y^4)(x^2+y^2)(x+y)(x-y)$

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P.52-53

因数分解を利用して計算しよう!

- 1 (1) 560 (2) 400
 (3) -60000 (4) 5000
 (5) 550 (6) 99.8
- 2 (1) 10000 (2) 3600
 (3) 1600 (4) 400
- 3 8の倍数

解き方

- 1 (1) $39^2 - 31^2 = (39+31)(39-31)$
 $= 70 \times 8 = 560$
- (3) $250^2 - 350^2 = (250+350)(250-350)$
 $= 600 \times (-100) = -60000$
- (4) $135^2 - 115^2 = (135+115)(135-115)$
 $= 250 \times 20 = 5000$
- (5) $30.5^2 - 19.5^2 = (30.5+19.5)(30.5-19.5)$
 $= 50 \times 11 = 550$
- (6) $9.99^2 - 0.01^2 = (9.99+0.01)(9.99-0.01)$
 $= 10 \times 9.98 = 99.8$
- 2 (1) $83^2 + 2 \times 83 \times 17 + 17^2 = (83+17)^2$
 $= 100^2 = 10000$
- (2) $74^2 - 2 \times 74 \times 14 + 14^2 = (74-14)^2$
 $= 60^2 = 3600$
- (3) $19^2 + 19 \times 42 + 21^2 = 19^2 + 19 \times (2 \times 21) + 21^2$
 $= 19^2 + 2 \times 19 \times 21 + 21^2$
 $= (19+21)^2 = 40^2 = 1600$
- (4) $35^2 + 15^2 - 70 \times 15$
 $= 35^2 - 70 \times 15 + 15^2 = 35^2 - 2 \times 35 \times 15 + 15^2$
 $= (35-15)^2 = 20^2 = 400$

- 3 連続する2つの奇数において、小さい方の奇数を $2n-1$ とおくと、大きい方の奇数は $(2n-1)+2=2n+1$ と表されます。大きい方の奇数の2乗から、小さい方の奇数の2乗を引くと、
 $(2n+1)^2 - (2n-1)^2$
 $= (4n^2+4n+1) - (4n^2-4n+1)$
 $= 8n$
 これは、 $8 \times (\text{ある整数})$ の形の式であるから、8の倍数です。つまり、連続する2つの奇数において、大きい方の奇数の2乗から、小さい方の奇数の2乗を引くと、8の倍数になります。

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P.54-55

確認問題④

- 1 (1) $a(a-7b)$ (2) $4x^2(2+y)$
 (3) $3a(8x-8y+z)$
 (4) $(3x-1)(a-2b)$
 (5) $(x-12)^2$ (6) $(a-\frac{1}{2})^2$
 (7) $(\frac{1}{2}x+\frac{1}{3})^2$ (8) $(a+13b)^2$
- 2 (1) $2ax(x-3)^2$ (2) $2a(3x+5y)^2$
 (3) $(3x+2)(3x-2)$
 (4) $3(3a+5b)(3a-5b)$
 (5) $3ax(y+3)(y-3)$
 (6) $(x^2+4)(x+2)(x-2)$

解き方

- 1 (1) $a^2 - 7ab = a \cdot a + a \cdot (-7b) = a(a-7b)$
 (2) $8x^2 + 4x^2y = 4x^2 \cdot 2 + 4x^2 \cdot y = 4x^2(2+y)$
 (4) $3x(a-2b) + 2b - a = 3x(a-2b) - (a-2b)$
 $= (3x-1)(a-2b)$
 (5) $x^2 - 24x + 144 = x^2 - 2 \cdot x \cdot 12 + 12^2 = (x-12)^2$
 (6) $a^2 - a + \frac{1}{4} = a^2 - 2 \cdot a \cdot \frac{1}{2} + (\frac{1}{2})^2 = (a-\frac{1}{2})^2$
 (7) $\frac{1}{4}x^2 + \frac{1}{3}x + \frac{1}{9}$
 $= (\frac{1}{2}x)^2 + 2 \cdot (\frac{1}{2}x) \cdot (\frac{1}{3}) + (\frac{1}{3})^2$
 $= (\frac{1}{2}x + \frac{1}{3})^2$
 (8) $a^2 + 26ab + 169b^2 = a^2 + 2 \cdot a \cdot 13b + (13b)^2$
 $= (a+13b)^2$
- 2 (1) $2ax^3 - 12ax^2 + 18ax = 2ax(x^2 - 6x + 9)$
 $= 2ax(x-3)^2$
 (2) $18ax^2 + 60axy + 50ay^2$
 $= 2a(9x^2 + 30xy + 25y^2)$
 $= 2a\{(3x)^2 + 2 \cdot 3x \cdot 5y + (5y)^2\}$
 $= 2a(3x+5y)^2$
 (3) $9x^2 - 4 = (3x)^2 - 2^2 = (3x+2)(3x-2)$
 (4) $27a^2 - 75b^2$
 $= 3(9a^2 - 25b^2) = 3\{(3a)^2 - (5b)^2\}$
 $= 3(3a+5b)(3a-5b)$
 (5) $3axy^2 - 27ax = 3ax(y^2 - 9) = 3ax(y^2 - 3^2)$
 $= 3ax(y+3)(y-3)$
 (6) $x^4 - 16 = (x^2)^2 - (2^2)^2 = (x^2+2^2)(x^2-2^2)$
 $= (x^2+4)(x+2)(x-2)$

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P.56-57

 $(x+a)(x+b)$ を使って因数分解しよう!①

- 1 (1) $(x+1)(x+3)$ (2) $(x+1)(x+8)$
 (3) $(x-3)(x-6)$ (4) $(x-2)(x-5)$
 (5) $(a+5)(a-10)$ (6) $(a-8)(a+10)$
- 2 (1) $(x+3y)(x+14y)$ (2) $(a+2b)(a+14b)$
 (3) $(x-3y)(x-4y)$ (4) $(x-a)(x+7a)$
 (5) $(x+5a)(x-7a)$ (6) $(a+4b)(a-5b)$
 (7) $(x-2y)(x+13y)$ (8) $(a-b)(a-2b)$

解き方

- 1 (1) 和が4、積が3となる2数は1と3であるから、 $x^2+4x+3=(x+1)(x+3)$
 (3) 和が-9、積が18となる2数は-3と-6であるから、 $x^2-9x+18=(x-3)(x-6)$
 (4) 和が-7、積が10となる2数は-2と-5であるから、 $x^2-7x+10=(x-2)(x-5)$
 (5) 和が-5、積が-50となる2数は5と-10であるから、
 $a^2-5a-50=(a+5)(a-10)$
 (6) 和が2、積が-80となる2数は-8と10であるから、 $a^2+2a-80=(a-8)(a+10)$
- 2 (1) 和が17y、積が42y²となる2式は3yと14yであるから、
 $x^2+17xy+42y^2=(x+3y)(x+14y)$
 (2) 和が16b、積が28b²となる2式は2bと14bであるから、
 $a^2+16ab+28b^2=(a+2b)(a+14b)$
 (3) 和が-7y、積が12y²となる2式は-3yと-4yであるから、
 $x^2-7xy+12y^2=(x-3y)(x-4y)$
 (4) 和が6a、積が-7a²となる2式は-aと7aであるから、
 $x^2+6ax-7a^2=(x-a)(x+7a)$
 (5) 和が-2a、積が-35a²となる2式は5aと-7aであるから、
 $x^2-2ax-35a^2=(x+5a)(x-7a)$
 (6) 和が-b、積が-20b²となる2式は4bと-5bであるから、
 $a^2-ab-20b^2=(a+4b)(a-5b)$
 (7) 和が11y、積が-26y²となる2式は-2yと13yであるから、
 $x^2+11xy-26y^2=(x-2y)(x+13y)$

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P.58-59

 $(x+a)(x+b)$ を使って因数分解しよう!②

- 1 (1) $2(x+3)(x+9)$ (2) $4(a+2)(a-3)$
 (3) $3(a-2)(a-6)$ (4) $4(x+1)(x+8)$
 (5) $2(a-1)(a-2)$ (6) $5(x-2)(x+5)$
- 2 (1) $a(x+2)(x-6)$ (2) $-3a(x+1)(x-4)$
 (3) $4(a+4b)(a+5b)$
 (4) $y(x-2y)(x+6y)$
 (5) $a(a-b)(a-12b)$
 (6) $3a(a-2b)(a-9b)$
 (7) $2x(yz+2)(yz-7)$
 (8) $2xy(x+y)(x+2y)$

解き方

- 1 (1) $2x^2+24x+54=2(x^2+12x+27)$
 $=2(x+3)(x+9)$
 (2) $4a^2-4a-24=4(a^2-a-6)$
 $=4(a+2)(a-3)$
 (3) $3a^2-24a+36=3(a^2-8a+12)$
 $=3(a-2)(a-6)$
 (5) $2a^2-6a+4=2(a^2-3a+2)$
 $=2(a-1)(a-2)$
 (6) $5x^2+15x-50=5(x^2+3x-10)$
 $=5(x-2)(x+5)$
- 2 (1) $ax^2-4ax-12a=a(x^2-4x-12)$
 $=a(x+2)(x-6)$
 (2) $-3ax^2+9ax+12a=-3a(x^2-3x-4)$
 $=-3a(x+1)(x-4)$
 (3) $4a^2+36ab+80b^2=4(a^2+9ab+20b^2)$
 $=4(a+4b)(a+5b)$
 (4) $x^2y+4xy^2-12y^3=y(x^2+4xy-12y^2)$
 $=y(x-2y)(x+6y)$
 (5) $a^3-13a^2b+12ab^2=a(a^2-13ab+12b^2)$
 $=a(a-b)(a-12b)$
 (6) $3a^3-33a^2b+54ab^2$
 $=3a(a^2-11ab+18b^2)$
 $=3a(a-2b)(a-9b)$
 (7) $2xy^2z^2-10xyz-28x$
 $=2x(y^2z^2-5yz-14)$
 $=2x(yz+2)(yz-7)$
 (8) $2x^3y+6x^2y^2+4xy^3$
 $=2xy(x^2+3xy+2y^2)$
 $=2xy(x+y)(x+2y)$

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P.60-61

 $(ax+b)(cx+d)$ を使って因数分解しよう!①

- 1 (1) $(5x+3)(x+1)$ (2) $(7x+3)(x+2)$
 (3) $(3x+7)(x-1)$ (4) $(2x+1)(x-3)$
 (5) $(2a-1)(a-2)$ (6) $(3a-2)(a+3)$

- 2 (1) $(3x+y)(2x+5y)$
 (2) $(2x-y)(2x+5y)$
 (3) $(2a+3b)(5a-3b)$
 (4) $(6a+b)(a+6b)$
 (5) $(4x+y)(3x-4y)$
 (6) $(2x-3y)(7x+5y)$
 (7) $(2a-5b)(a-b)$
 (8) $(3x-5y)(2x-3y)$

解き方

1 (1) $5x^2+8x+3 = (5x+3)(x+1)$ $\begin{array}{r} 5 \quad 3 \cdots 3 \\ 1 \quad \quad 1 \cdots 5 \\ \hline 8 \end{array}$

(3) $3x^2+4x-7 = (3x+7)(x-1)$ $\begin{array}{r} 3 \quad 7 \cdots 7 \\ 1 \quad -1 \cdots -3 \\ \hline 4 \end{array}$

(5) $2a^2-5a+2 = (2a-1)(a-2)$ $\begin{array}{r} 2 \quad -1 \cdots -1 \\ 1 \quad -2 \cdots -4 \\ \hline -5 \end{array}$

(6) $3a^2+7a-6 = (3a-2)(a+3)$ $\begin{array}{r} 3 \quad -2 \cdots -2 \\ 1 \quad 3 \cdots 9 \\ \hline 7 \end{array}$

2 (1) $6x^2+17xy+5y^2 = (3x+y)(2x+5y)$ $\begin{array}{r} 3 \quad y \cdots 2y \\ 2 \quad 5y \cdots 15y \\ \hline 17y \end{array}$

(3) $10a^2+9ab-9b^2 = (2a+3b)(5a-3b)$ $\begin{array}{r} 2 \quad 3b \cdots 15b \\ 5 \quad -3b \cdots -6b \\ \hline 9b \end{array}$

(5) $12x^2-13xy-4y^2 = (4x+y)(3x-4y)$ $\begin{array}{r} 4 \quad y \cdots 3y \\ 3 \quad -4y \cdots -16y \\ \hline -13y \end{array}$

(6) $14x^2-11xy-15y^2 = (2x-3y)(7x+5y)$ $\begin{array}{r} 2 \quad -3y \cdots -21y \\ 7 \quad 5y \cdots 10y \\ \hline -11y \end{array}$

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P.62-63

 $(ax+b)(cx+d)$ を使って因数分解しよう!②

- 1 (1) $3(3x+2)(x+1)$ (2) $2(2x-1)(x-4)$
 (3) $5(3x+1)(x+3)$ (4) $3(6a+1)(2a-3)$
 (5) $3(3x+2)(x-1)$ (6) $2(3a-2)(2a+3)$

- 2 (1) $3(2a+3b)(a-2b)$
 (2) $2(4x-3y)(2x+y)$
 (3) $5(3a-4b)(2a+3b)$
 (4) $6(3xy-1)(2xy+1)$
 (5) $3x(2x+1)(x-3)$
 (6) $-2a(x+2)(6x-5)$
 (7) $y(2x-11y)(3x+7y)$
 (8) $2a(3a-b)(a-2b)$

解き方

1 (1) $9x^2+15x+6 = 3(3x^2+5x+2) = 3(3x+2)(x+1)$ $\begin{array}{r} 3 \quad 2 \cdots 2 \\ 1 \quad 1 \cdots 3 \\ \hline 5 \end{array}$

(2) $4x^2-18x+8 = 2(2x^2-9x+4) = 2(2x-1)(x-4)$ $\begin{array}{r} 2 \quad -1 \cdots -1 \\ 1 \quad -4 \cdots -8 \\ \hline -9 \end{array}$

(4) $36a^2-48a-9 = 3(12a^2-16a-3) = 3(6a+1)(2a-3)$ $\begin{array}{r} 6 \quad 1 \cdots 2 \\ 2 \quad -3 \cdots -18 \\ \hline -16 \end{array}$

(6) $12a^2+10a-12 = 2(6a^2+5a-6) = 2(3a-2)(2a+3)$ $\begin{array}{r} 3 \quad -2 \cdots -4 \\ 2 \quad 3 \cdots 9 \\ \hline 5 \end{array}$

2 (1) $6a^2-3ab-18b^2 = 3(2a^2-ab-6b^2) = 3(2a+3b)(a-2b)$ $\begin{array}{r} 2 \quad 3b \cdots 3b \\ 1 \quad -2b \cdots -4b \\ \hline -b \end{array}$

(3) $30a^2+5ab-60b^2 = 5(6a^2+ab-12b^2) = 5(3a-4b)(2a+3b)$ $\begin{array}{r} 3 \quad -4b \cdots -8b \\ 2 \quad 3b \cdots 9b \\ \hline b \end{array}$

(6) $-12ax^2-14ax+20a = -2a(6x^2+7x-10) = -2a(x+2)(6x-5)$ $\begin{array}{r} 1 \quad 2 \cdots 12 \\ 6 \quad -5 \cdots -5 \\ \hline 7 \end{array}$

(7) $6x^2y-19xy^2-77y^3 = y(6x^2-19xy-77y^2) = y(2x-11y)(3x+7y)$ $\begin{array}{r} 2 \quad -11y \cdots -33y \\ 3 \quad 7y \cdots 14y \\ \hline -19y \end{array}$

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P.64-65

工夫して因数分解しよう!①

- 1 (1) $(x+2)(x+8)$
 (2) $(x-y-1)(x-y-2)$
 (3) $(a+b+1)(a+b-5)$
 (4) $(3x-3y+8)(4x-4y-3)$
 (5) $(2a+2b-3)(4a+4b+5)$
 (6) $(x+1)(x+2)(x-2)(x+5)$

- 2 (1) $(a^2+a-1)(a+2)(a-1)$
 (2) $(x-1)(x-2)(x+1)(x-4)$
 (3) $(x-1)(x+4)(x^2+3x+6)$
 (4) $(a^2+8a+10)(a+2)(a+6)$

解き方

1 (4) $x-y=A$ とおくと、
 与式 $=12A^2+23A-24$ $\begin{array}{r} 3 \times 8 \dots 32 \\ 4 \times -3 \dots -9 \\ \hline 23 \end{array}$
 $= (3A+8)(4A-3)$

$= \{3(x-y)+8\}\{4(x-y)-3\}$
 $= (3x-3y+8)(4x-4y-3)$

(5) $a+b=A$ とおくと、
 与式 $=8A^2-2A-15$ $\begin{array}{r} 2 \times -3 \dots -12 \\ 4 \times 5 \dots 10 \\ \hline -2 \end{array}$
 $= (2A-3)(4A+5)$

$= \{2(a+b)-3\}\{4(a+b)+5\}$
 $= (2a+2b-3)(4a+4b+5)$

(6) $x^2+3x=A$ とおくと、
 与式 $=A^2-8A-20=(A+2)(A-10)$
 $= \{(x^2+3x)+2\}\{(x^2+3x)-10\}$
 $= (x^2+3x+2)(x^2+3x-10)$
 $= (x+1)(x+2)(x-2)(x+5)$

2 (1) $(a^2+a+2)(a^2+a-5)+12$
 $= (A+2)(A-5)+12$ ← a^2+a を A とおく
 $= A^2-3A-10+12=A^2-3A+2$
 $= (A-1)(A-2)$
 $= \{(a^2+a)-1\}\{(a^2+a)-2\}$
 $= (a^2+a-1)(a^2+a-2)$
 $= (a^2+a-1)(a+2)(a-1)$

(3) $x(x+3)(x+1)(x+2)-24$
 $= (x^2+3x)(x^2+3x+2)-24$
 $= A(A+2)-24$ ← x^2+3x を A とおく
 $= A^2+2A-24=(A-4)(A+6)$
 $= (x^2+3x-4)(x^2+3x+6)$
 $= (x-1)(x+4)(x^2+3x+6)$

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P.66-67

工夫して因数分解しよう!②

- 1 (1) $(y-1)(x+y)$ (2) $(x+4)(y+x-3)$
 (3) $(a-1)(ab-1)$ (4) $(a-b)(c+a+b)$
 (5) $(a+b)(a-b)(-c+b)$
 (6) $(2a-c)(b+2a-c)$
 2 (1) $(x-y+1)(x-y-2)$
 (2) $(a-b+1)(a-b+2)$
 (3) $(2x+y-3)(x-y-2)$
 (4) $(x-4y+2)(x-y-1)$

解き方

1 (1) 与式 $=xy-x+y^2-y=(y-1)x+(y-1)y$
 $= (y-1)(x+y)$

(2) x, y について次数はそれぞれ、2、1であるから、次数の低い y について整理します。
 与式 $=xy+4y+x^2+x-12$
 $= (x+4)y+(x+4)(x-3)$
 $= (x+4)(y+x-3)$

(3) a, b について次数はそれぞれ、2、1であるから、次数の低い b について整理します。
 与式 $=a^2b-ab-a+1=(a^2-a)b-a+1$
 $= a(a-1)b-(a-1)$
 $= (a-1)(ab-1)$

(4) a, b, c について次数はそれぞれ、2、2、1であるから、次数の低い c について整理します。
 与式 $=ac-bc+a^2-b^2$
 $= (a-b)c+(a-b)(a+b)$
 $= (a-b)(c+a+b)$

2 (1) x, y ともに次数は2であるから、例えば、 x について整理します。

与式 $=x^2-2xy-x+y^2+y-2$
 $= x^2-(2y+1)x+(y^2+y-2)$
 $= x^2-(2y+1)x+(y-1)(y+2)$
 $= \{x-(y-1)\}\{x-(y+2)\}$
 $= (x-y+1)(x-y-2)$

(3) x, y ともに次数は2であるから、例えば、 x について整理します。

与式 $=2x^2-xy-7x-y^2+y+6$
 $= 2x^2-(y+7)x-(y^2-y-6)$
 $= 2x^2-(y+7)x-(y+2)(y-3)$
 $= \{2x+(y-3)\}\{x-(y+2)\}$
 $= (2x+y-3)(x-y-2)$

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P.68-69

確認問題⑤

- 1 (1) $(x-3)(x-6)$ (2) $(x-2)(x+8)$
 (3) $(x+6)(x-10)$ (4) $(x-4)(x-15)$
 (5) $3(x-3)(x+4)$ (6) $4(x+2y)(x-4y)$
- 2 (1) $(2x-3)(3x+5)$ (2) $(4x-5)(x-3)$
 (3) $(3a+2)(2a-3)$ (4) $(2a+b)(a-6b)$
- 3 (1) $6(2x-1)(x+1)$
 (2) $4(3x+2)(x-2)$
- 4 (1) $(x+5)(x-1)$
 (2) $x(x-1)(x^2-x-14)$

解き方

- 1 (1) $x^2-9x+18$
 $=x^2+(-3-6)x+(-3)\cdot(-6)$
 $= (x-3)(x-6)$
- (3) $x^2-4x-60=x^2+(6-10)x+6\cdot(-10)$
 $= (x+6)(x-10)$
- (5) $3x^2+3x-36=3(x^2+x-12)$
 $= 3(x-3)(x+4)$
- 2 (1) $6x^2+x-15$ $\begin{array}{r} 2 \quad -3 \dots -9 \\ 3 \quad 5 \dots 10 \\ \hline \end{array}$
 $= (2x-3)(3x+5)$
- (4) $2a^2-11ab-6b^2$ $\begin{array}{r} 2 \quad b \dots b \\ 1 \quad -6b \dots -12b \\ \hline \quad \quad -11b \end{array}$
 $= (2a+b)(a-6b)$
- 3 (1) $12x^2+6x-6$ $\begin{array}{r} 2 \quad -1 \dots -1 \\ 1 \quad 1 \dots 2 \\ \hline \end{array}$
 $= 6(2x^2+x-1)$
 $= 6(2x-1)(x+1)$
- 4 (1) $x+3=A$ とおくと、
 与式 $= A^2-2A-8=(A+2)(A-4)$
 $= \{(x+3)+2\}\{(x+3)-4\}$
 $= (x+5)(x-1)$
- (2) $(x+1)(x+3)(x-4)(x-2)-24$
 $= (x+1)(x-2)(x+3)(x-4)-24$
 $= (x^2-x-2)(x^2-x-12)-24$
 $= (x^2-x)^2-14(x^2-x)+24-24$
 $= (x^2-x)^2-14(x^2-x)$
 $= (x^2-x)(x^2-x-14)$
 $= x(x-1)(x^2-x-14)$

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P.70-71

 $(a+b)(a^2-ab+b^2)$ 、 $(a-b)(a^2+ab+b^2)$
 を使って因数分解しよう!①

- 1 (1) $(a+1)(a^2-a+1)$
 (2) $(x-2)(x^2+2x+4)$
 (3) $(a+4)(a^2-4a+16)$
 (4) $(1-x)(1+x+x^2)$
 (5) $(2a+3)(4a^2-6a+9)$
 (6) $(3x-4)(9x^2+12x+16)$
- 2 (1) $(a-3b)(a^2+3ab+9b^2)$
 (2) $(x+y)(x^2-xy+y^2)$
 (3) $(a+4b)(a^2-4ab+16b^2)$
 (4) $(x-5y)(x^2+5xy+25y^2)$
 (5) $(5a-2b)(25a^2+10ab+4b^2)$
 (6) $(4x+3y)(16x^2-12xy+9y^2)$
 (7) $(xy-1)(x^2y^2+xy+1)$
 (8) $(ab+c)(a^2b^2-abc+c^2)$

解き方

- 1 (1) $a^3+1=a^3+1^3$
 $= (a+1)(a^2-a\cdot 1+1^2)=(a+1)(a^2-a+1)$
- (4) $1-x^3=1^3-x^3=(1-x)(1^2+1\cdot x+x^2)$
 $= (1-x)(1+x+x^2)$
- (5) $8a^3+27=(2a)^3+3^3$
 $= (2a+3)\{(2a)^2-(2a)\cdot 3+3^2\}$
 $= (2a+3)(4a^2-6a+9)$
- 2 (1) $a^3-27b^3=a^3-(3b)^3$
 $= (a-3b)\{a^2+a\cdot(3b)+(3b)^2\}$
 $= (a-3b)(a^2+3ab+9b^2)$
- (2) $x^3+y^3=(x+y)(x^2-x\cdot y+y^2)$
 $= (x+y)(x^2-xy+y^2)$
- (3) $a^3+64b^3=a^3+(4b)^3$
 $= (a+4b)\{a^2-a\cdot(4b)+(4b)^2\}$
 $= (a+4b)(a^2-4ab+16b^2)$
- (4) $x^3-125y^3=x^3-(5y)^3$
 $= (x-5y)\{x^2+x\cdot(5y)+(5y)^2\}$
 $= (x-5y)(x^2+5xy+25y^2)$
- (7) $x^3y^3-1=(xy)^3-1^3$
 $= (xy-1)\{(xy)^2+(xy)\cdot 1+1^2\}$
 $= (xy-1)(x^2y^2+xy+1)$
- (8) $a^3b^3+c^3=(ab)^3+c^3$
 $= (ab+c)\{(ab)^2-(ab)\cdot c+c^2\}$
 $= (ab+c)(a^2b^2-abc+c^2)$

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P.72-73

 $(a+b)(a^2-ab+b^2)$ 、 $(a-b)(a^2+ab+b^2)$ を使って因数分解しよう!②

- 1 (1) $2(a+3b)(a^2-3ab+9b^2)$
 (2) $3(x-2y)(x^2+2xy+4y^2)$
 (3) $4(a-b)(a^2+ab+b^2)$
 (4) $2(4x+y)(16x^2-4xy+y^2)$
 (5) $3(2a+3b)(4a^2-6ab+9b^2)$
 (6) $2(5x-2y)(25x^2+10xy+4y^2)$

- 2 (1) $\frac{1}{2}\left(\frac{1}{2}x-y\right)\left(\frac{1}{4}x^2+\frac{1}{2}xy+y^2\right)$
 (2) $a(5a-2b)(25a^2+10ab+4b^2)$
 (3) $ab(7-ab)(49+7ab+a^2b^2)$
 (4) $a^2b^2(a-b)(a^2+ab+b^2)$
 (5) $2z^3(2xyz+1)(4x^2y^2z^2-2xyz+1)$

解き方

1 (4) $128x^3+2y^3=2(64x^3+y^3)=2\{(4x)^3+y^3\}$
 $=2(4x+y)\{(4x)^2-(4x)\cdot y+y^2\}$
 $=2(4x+y)(16x^2-4xy+y^2)$

(6) $250x^3-16y^3=2(125x^3-8y^3)$
 $=2\{(5x)^3-(2y)^3\}$
 $=2(5x-2y)\{(5x)^2+(5x)\cdot(2y)+(2y)^2\}$
 $=2(5x-2y)(25x^2+10xy+4y^2)$

2 (1) $\frac{1}{16}x^3-\frac{1}{2}y^3=\frac{1}{2}\left(\frac{1}{8}x^3-y^3\right)$
 $=\frac{1}{2}\left\{\left(\frac{1}{2}x\right)^3-y^3\right\}$
 $=\frac{1}{2}\left(\frac{1}{2}x-y\right)\left\{\left(\frac{1}{2}x\right)^2+\left(\frac{1}{2}x\right)\cdot y+y^2\right\}$
 $=\frac{1}{2}\left(\frac{1}{2}x-y\right)\left(\frac{1}{4}x^2+\frac{1}{2}xy+y^2\right)$

(2) $125a^4-8ab^3=a(125a^3-8b^3)$
 $=a\{(5a)^3-(2b)^3\}$
 $=a(5a-2b)\{(5a)^2+(5a)\cdot(2b)+(2b)^2\}$
 $=a(5a-2b)(25a^2+10ab+4b^2)$

(3) $343ab-a^4b^4=ab(343-a^3b^3)$
 $=ab\{7^3-(ab)^3\}$
 $=ab(7-ab)\{7^2+7\cdot(ab)+(ab)^2\}$
 $=ab(7-ab)(49+7ab+a^2b^2)$

(5) $16x^3y^3z^6+2z^3=2z^3(8x^3y^3z^3+1)$
 $=2z^3\{(2xyz)^3+1^3\}$
 $=2z^3(2xyz+1)\{(2xyz)^2-(2xyz)\cdot 1+1^2\}$
 $=2z^3(2xyz+1)(4x^2y^2z^2-2xyz+1)$

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P.74-75

工夫して因数分解しよう!③

- 1 (1) $(x+3)(x-3)(x^2-2)$
 (2) $(3a+2)(3a-2)(2a^2+1)$
 (3) $2(a+4)(a-4)(a^2+1)$
 (4) $(2x+1)(2x-1)(x+2)(x-2)$
 (5) $2(x+5)(x-5)(x^2+2)$
 (6) $3x(x+2)(x-2)(x^2+5)$
- 2 (1) $(x+1)(x^2-x+1)(x-1)(x^2+x+1)$
 (2) $(2a+b)(4a^2-2ab+b^2)$
 $\times(2a-b)(4a^2+2ab+b^2)$
 (3) $(a+bc)(a^2-abc+b^2c^2)$
 $\times(a-bc)(a^2+abc+b^2c^2)$
 (4) $(x+2)(x^2-2x+4)(x^3-2)$
 (5) $(x-3)(x^2+3x+9)(x+1)(x^2-x+1)$

解き方

1 (2) $a^2=A$ とおくと、
 与式 $=18A^2+A-4$ $\begin{array}{r} 9 \quad -4 \dots -8 \\ 2 \quad \quad \quad 1 \dots 9 \\ \hline \quad \quad \quad \quad \quad 1 \end{array}$
 $= (9A-4)(2A+1)$
 $= (9a^2-4)(2a^2+1)$
 $= (3a+2)(3a-2)(2a^2+1)$

(4) $x^2=A$ とおくと、
 与式 $=4A^2-17A+4=(4A-1)(A-4)$
 $= (4x^2-1)(x^2-4)$
 $= (2x+1)(2x-1)(x+2)(x-2)$

2 (1) $x^6-1=(x^3)^2-1^2=(x^3+1)(x^3-1)$
 $= (x+1)(x^2-x+1)(x-1)(x^2+x+1)$

(2) $64a^6-b^6=(8a^3)^2-(b^3)^2$
 $= (8a^3+b^3)(8a^3-b^3)$
 $= (2a+b)(4a^2-2ab+b^2)$
 $\times(2a-b)(4a^2+2ab+b^2)$

(3) $a^6-b^6c^6=(a^3)^2-(b^3c^3)^2$
 $= (a^3+b^3c^3)(a^3-b^3c^3)$
 $= (a+bc)(a^2-abc+b^2c^2)$
 $\times(a-bc)(a^2+abc+b^2c^2)$

(4) $x^6+6x^3-16=(x^3)^2+6x^3-16$
 $= (x^3+8)(x^3-2)$
 $= (x+2)(x^2-2x+4)(x^3-2)$

(5) $x^6-26x^3-27=(x^3)^2-26x^3-27$
 $= (x^3-27)(x^3+1)$
 $= (x-3)(x^2+3x+9)(x+1)(x^2-x+1)$

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P.76-77

工夫して因数分解しよう!④

- 1 (1) $a(a^2-3ab+3b^2)$
 (2) $(3x+y)(9x^2-12xy+7y^2)$
 (3) $2b(3a^2+b^2)$
 (4) $(4x+5y)(4x^2+10xy+13y^2)$

- 2 (1) $(x^2+x+2)(x^2-x+2)$
 (2) $(a^2+2a+2)(a^2-2a+2)$
 (3) $(x^2+2x+3)(x^2-2x+3)$
 (4) $(a^2+3a+1)(a^2-3a+1)$
 (5) $(a^2+ab+b^2)(a^2-ab+b^2)$
 (6) $(x^2+4x+8)(x^2-4x+8)$

解き方

- 1 (1) $(a-b)^3+b^3$
 $=\{(a-b)+b\}\{(a-b)^2-(a-b)\cdot b+b^2\}$
 $=a(a^2-2ab+b^2-ab+b^2+b^2)$
 $=a(a^2-3ab+3b^2)$
- (2) $(3x-y)^3+8y^3=(3x-y)^3+(2y)^3$
 $=\{(3x-y)+2y\}$
 $\times\{(3x-y)^2-(3x-y)\cdot(2y)+(2y)^2\}$
 $=\{(3x+y)(9x^2-6xy+y^2-6xy+2y^2+4y^2)\}$
 $=\{(3x+y)(9x^2-12xy+7y^2)\}$
- (3) $(a+b)^3-(a-b)^3$
 $=\{(a+b)-(a-b)\}$
 $\times\{(a+b)^2+(a+b)\cdot(a-b)+(a-b)^2\}$
 $=2b(a^2+2ab+b^2+a^2-b^2+a^2-2ab+b^2)$
 $=2b(3a^2+b^2)$
- 2 (1) $x^4+3x^2+4=(x^4+4x^2+4)-x^2$
 $=\{(x^2+2)^2-x^2\}=(x^2+2+x)(x^2+2-x)$
 $=\{(x^2+x+2)(x^2-x+2)\}$
- (2) $a^4+4=(a^4+4a^2+4)-4a^2$
 $=\{(a^2+2)^2-(2a)^2\}$
 $=\{(a^2+2+2a)(a^2+2-2a)\}$
 $=\{(a^2+2a+2)(a^2-2a+2)\}$
- (4) $a^4-7a^2+1=(a^4+2a^2+1)-9a^2$
 $=\{(a^2+1)^2-(3a)^2\}$
 $=\{(a^2+1+3a)(a^2+1-3a)\}$
 $=\{(a^2+3a+1)(a^2-3a+1)\}$
- (5) $a^4+a^2b^2+b^4=(a^4+2a^2b^2+b^4)-a^2b^2$
 $=\{(a^2+b^2)^2-(ab)^2\}$
 $=\{(a^2+b^2+ab)(a^2+b^2-ab)\}$
 $=\{(a^2+ab+b^2)(a^2-ab+b^2)\}$

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P.78-79

確認問題⑥

- 1 (1) $(a-4)(a^2+4a+16)$
 (2) $(x+1)(x^2-x+1)$
 (3) $(ab+7)(a^2b^2-7ab+49)$
 (4) $8(x-3y)(x^2+3xy+9y^2)$
 (5) $2(a+4)(a^2-4a+16)$
 (6) $3(x-3y)(x^2+3xy+9y^2)$
- 2 (1) $(a+2)(a^2-2a+4)(a-2)(a^2+2a+4)$
 (2) $(2x^2-2x+1)(2x^2+2x+1)$
- 3 (1) $(x-1)(x+2)(x^4+2x^3+3x^2+2x+4)$
 (2) $x(x^2+3xy+3y^2)$
 (3) $(a-b)(7a^2+13ab+7b^2)$
 (4) $(x+y)(3x+3y-1)$
 $\times(9x^2+18xy+9y^2+3x+3y+1)$

解き方

- 1 (1) $a^3-64=a^3-4^3$
 $=\{(a-4)(a^2+a\cdot 4+4^2)\}$
 $=\{(a-4)(a^2+4a+16)\}$
- (3) $a^3b^3+343=(ab)^3+7^3$
 $=\{(ab+7)\{(ab)^2-(ab)\cdot 7+7^2\}\}$
 $=\{(ab+7)(a^2b^2-7ab+49)\}$
- (4) $8x^3-216y^3=8(x^3-27y^3)$
 $=8\{x^3-(3y)^3\}$
 $=8(x-3y)\{x^2+x\cdot(3y)+9y^2\}$
 $=8(x-3y)(x^2+3xy+9y^2)$
- 2 (1) $a^6-64=(a^3)^2-8^2=(a^3+8)(a^3-8)$
 $=\{(a^3+2^3)(a^3-2^3)\}$
 $=\{(a+2)(a^2-2a+4)(a-2)(a^2+2a+4)\}$
- 3 (1) $(x^2+x)^3-8=(x^2+x)^3-2^3$
 $=\{(x^2+x)-2\}\{(x^2+x)^2+2(x^2+x)+4\}$
 $=\{(x^2+x-2)(x^4+2x^3+3x^2+2x+4)\}$
 $=\{(x-1)(x+2)(x^4+2x^3+3x^2+2x+4)\}$
- (3) $(2a+b)^3-(a+2b)^3$
 $=\{(2a+b)-(a+2b)\}$
 $\times\{(2a+b)^2+(2a+b)(a+2b)+(a+2b)^2\}$
 $=\{(a-b)(4a^2+4ab+b^2+2a^2+5ab$
 $+2b^2+a^2+4ab+4b^2)\}$
 $=\{(a-b)(7a^2+13ab+7b^2)\}$

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P.86-87

平方根を求めよう!

- 1 (1) ± 15 (2) ± 0.9 (3) $\pm \frac{7}{4}$
 (4) ± 1000
- 2 (1) $\pm\sqrt{5}$ (2) $\pm\sqrt{123}$ (3) $\pm\sqrt{0.4}$
 (4) $\pm\sqrt{\frac{14}{5}}$
- 3 (1) 12 (2) -14 (3) 0.01
 (4) 2
- 4 (1) $\sqrt{15} < 4$ (2) $\sqrt{10} < 3.2$
 (3) $\sqrt{31} > 5.5$ (4) $\sqrt{0.05} > \frac{1}{5}$
- 5 (1) 33 (2) 20
- 6 13, 14, 15, 16, 17, 18, 19, 20

解き方

- 1 (1) 225の平方根は、 $15^2=225$ 、
 $(-15)^2=225$ であるから、 ± 15
 (2) 0.81の平方根は、 $0.9^2=0.81$ 、
 $(-0.9)^2=0.81$ であるから、 ± 0.9
 (3) $\frac{49}{16}$ の平方根は、 $(\frac{7}{4})^2 = \frac{49}{16}$ 、
 $(-\frac{7}{4})^2 = \frac{49}{16}$ であるから、 $\pm \frac{7}{4}$
- 2 $a > 0$ のとき、 a の平方根は、正と負の2つあり、正の方を \sqrt{a} 、負の方を $-\sqrt{a}$ で表します。
- 3 (1) $\sqrt{144} = \sqrt{12^2} = 12$
 (2) $-\sqrt{196} = -\sqrt{14^2} = -14$
 (3) $\sqrt{0.0001} = \sqrt{0.01^2} = 0.01$
 (4) $\sqrt{\frac{(-6)^2}{9}} = \sqrt{\frac{36}{9}} = \sqrt{4} = 2$
- 4 (1) $(\sqrt{15})^2 = 15$ 、 $4^2 = 16$ 、 $(\sqrt{15})^2 < 4^2$ であるから、 $\sqrt{15} < 4$
 (別解) $4 = \sqrt{16}$ 、 $15 < 16$ であるから、
 $\sqrt{15} < \sqrt{16}$ よって、 $\sqrt{15} < 4$
- 5 (1) $5.7 < \sqrt{x} < 6.2$ であるから、 $5.7^2 < x < 6.2^2$
 $5.7^2 = 32.49$ 、 $6.2^2 = 38.44$ より、求める値は33です。
- 6 $3.6 < \sqrt{x} < 4.5$ であるから、 $3.6^2 < x < 4.5^2$
 $3.6^2 = 12.96$ 、 $4.5^2 = 20.25$ より、求める値は13, 14, 15, 16, 17, 18, 19, 20です。

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P.88-89

実数って何?

- 1 (1) 0.1125 (2) $0.\dot{1}2\dot{3}$ (3) $0.\dot{2}8571\dot{4}$
- 2 (1) $\sqrt{16}$ 、 $-\frac{91}{13}$ 、 0
 (2) $\frac{1}{2}$ 、 2.3 、 $\frac{231}{6}$ 、 $-\frac{14}{35}$ (3) $\sqrt{\frac{16}{49}}$ 、 $\frac{5}{13}$
 (4) π 、 $\sqrt{12}$ 、 $\sqrt{45}$ 、 $-\sqrt{10}$
- 3 (1) $\frac{7}{9}$ (2) $\frac{7}{33}$ (3) $\frac{24}{37}$ (4) $\frac{556}{495}$

解き方

- 1 (2) $\frac{41}{333} = 0.123123123\cdots = 0.\dot{1}2\dot{3}$
 (3) $\frac{2}{7} = 0.285714285714\cdots = 0.\dot{2}8571\dot{4}$
- 2 $\frac{1}{2} = 0.5$ (有限小数)、 $\sqrt{16} = 4$ (整数)、
 2.3 (有限小数)、 π (無理数)、
 $\sqrt{12} = 2\sqrt{3}$ (無理数)、
 $\sqrt{\frac{16}{49}} = \frac{4}{7} = 0.\dot{5}7142\dot{8}$ (循環小数)、
 $-\frac{91}{13} = -7$ (整数)、 $\sqrt{45} = 3\sqrt{5}$ (無理数)、
 $\frac{231}{6} = 38.5$ (有限小数)、
 $-\sqrt{10}$ (無理数)、 0 (整数)、
 $-\frac{14}{35} = -0.4$ (有限小数)、
 $\frac{5}{13} = 0.\dot{3}8461\dot{5}$ (循環小数)
- 3 (1) $x = 0.\dot{7}$ とおく。
 $10x = 7.7777777\cdots$
 $-) \quad x = 0.7777777\cdots$
 $\hline 9x = 7$
 よって、 $x = \frac{7}{9}$
- (4) $x = 0.1\dot{2}\dot{3}$ とおく。
 $1000x = 123.23232323\cdots$
 $-) \quad 10x = 1.23232323\cdots$
 $\hline 990x = 122$
 よって、 $x = \frac{122}{990} = \frac{61}{495}$
 したがって、 $1.1\dot{2}\dot{3} = 1 + \frac{61}{495} = \frac{556}{495}$

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P.90-91

絶対値って何？

- 1 (1) 4と-4
 (2) 整数…-4、-3、-2、-1、0、1、2、3、4
 自然数…1、2、3、4
 (3) 整数…-3、-2、-1、0、1、2、3
 自然数…1、2、3

- 2 (1) 9 (2) $\frac{4}{3}$ (3) $-3+\pi$
 (4) $-\sqrt{5}+3$

- 3 (1) 7 (2) 2 (3) 5

- 4 (1) 9 (2) 5 (3) 3 (4) 11

解き方

- 1 (1) $a>0$ のとき、 $|x|=a$ となる数は2つあり、 a と $-a$ です。

(2) 絶対値が4以下の整数に、4と-4は含むことに注意しましょう。また、0は自然数ではなく、整数であることにも注意しましょう。

(3) 絶対値が4より小さい整数に、4と-4は含まないことに注意しましょう。また、絶対値が4より小さい自然数に、4は含まないことに注意しましょう。

- 2 $a>0$ のとき、 $|a|=a$ 、 $|-a|=a$ となることに注意しましょう。絶対値はその数から符号を取り去ったものと考えてもよいです。

(3) $\pi=3.14\dots$ なので、 $3-\pi<0$
 よって、 $|3-\pi|=- (3-\pi)=-3+\pi$

(4) $4<5<9$ より、 $2<\sqrt{5}<3$ なので、 $\sqrt{5}-3<0$
 $|\sqrt{5}-3|=-(\sqrt{5}-3)=-\sqrt{5}+3$

- 3 2点 a 、 b 間の距離は、 $|b-a|$ で求められます。

- (1) $|10-3|=7$
 (2) $|-5-(-3)|=|-5+3|=|-2|=2$
 (3) $|-4-1|=|-5|=5$

- 4 (1) $P=|3\cdot(-1)-2|+|3-(-1)|$
 $=|-5|+|4|=5+4=9$

(2) $P=|3\cdot 0-2|+|3-0|$
 $=|-2|+|3|=2+3=5$

(3) $P=|3\cdot 1-2|+|3-1|$
 $=|1|+|2|=1+2=3$

(4) $P=|3\cdot 4-2|+|3-4|$
 $=|10|+|-1|=10+1=11$

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P.92-93

複素数って何？

- 1 i と $-i$

- 2 (1) 実部…-5、虚部…3

(2) 実部… $-\frac{1}{3}$ 、虚部… $\frac{1}{2}$

(3) 実部…2、虚部…4

(4) 実部…0、虚部…-7

- 3 (1) $x=4$ 、 $y=-2$ (2) $x=\frac{1}{2}$ 、 $y=-3$

(3) $x=-4$ 、 $y=2$ (4) $x=2$ 、 $y=-2$

(5) $x=-\frac{4}{3}$ 、 $y=-\frac{23}{3}$

(6) $x=1$ 、 $y=-2$

解き方

- 2 (1) 虚部は $3i$ ではなく 3 です。注意しましょう。

(2) $\frac{3i-2}{6}=-\frac{1}{3}+\frac{1}{2}i$ です。

(3) $i^2+4i+3=-1+4i+3=2+4i$ です。

(4) $-7i=0-7i$ です。

- 3 (1) 左辺を $a+bi$ の形に変形すると、

$$(x-4)+(-2-y)i=0$$

$x-4$ 、 $-2-y$ は実数であるから、

$$x-4=0 \dots \textcircled{1}, -2-y=0 \dots \textcircled{2}$$

①、②より、 $x=4$ 、 $y=-2$

- (3) 左辺を $a+bi$ の形に変形すると、

$$(x+4)+(x+2y)i=0$$

$x+4$ 、 $x+2y$ は実数であるから、

$$x+4=0 \dots \textcircled{1}, x+2y=0 \dots \textcircled{2}$$

①、②より、 $x=-4$ 、 $y=2$

- (5) 左辺を $a+bi$ の形に変形すると、

$$(2x-y-5)+(3x+4)i=0$$

$2x-y-5$ 、 $3x+4$ は実数であるから、

$$2x-y-5=0 \dots \textcircled{1}, 3x+4=0 \dots \textcircled{2}$$

①、②より、 $x=-\frac{4}{3}$ 、 $y=-\frac{23}{3}$

- (6) 左辺を $a+bi$ の形に変形すると、

$$(x+2y+3)+(3x+y-1)i=0$$

$x+2y+3$ 、 $3x+y-1$ は実数であるから、

$$x+2y+3=0 \dots \textcircled{1}, 3x+y-1=0 \dots \textcircled{2}$$

①、②より、 $x=1$ 、 $y=-2$

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P.94-95

複素数の足し算、引き算、掛け算をしよう!

- 1 (1) $1+8i$ (2) $8i$
 (3) $8+10i$ (4) $-1+4i$
- 2 (1) $5+5i$ (2) $-3-4i$
 (3) $-23+2i$ (4) $-243i$
 (5) $-2+2i$ (6) 20
- 3 $2\sqrt{3}-2\sqrt{3}i, -2\sqrt{3}+2\sqrt{3}i$

解き方

- 1 (2) $(5+4i)-(5-4i)=5+4i-5+4i$
 $= (5-5)+(4+4)i=8i$
- (3) $4(3+2i)+2(i-2)=12+8i+2i-4$
 $= (12-4)+(8+2)i=8+10i$
- (4) $3(1-2i)-2(2-5i)=3-6i-4+10i$
 $= (3-4)+(-6+10)i=-1+4i$
- 2 (1) $(1+2i)(3-i)=3-i+6i-2i^2$
 $= 3-i+6i+2=5+5i$
- (2) $(1-2i)^2=1-4i+4i^2$
 $= 1-4i-4=-3-4i$
- (4) $(-3i)^5=(-3i)^2(-3i)^2(-3i)$
 $= 9i^2 \cdot 9i^2 \cdot (-3i)$
 $= (-9) \cdot (-9) \cdot (-3i)=-243i$
- (5) $(1+i)^3=1^3+3 \cdot 1^2 \cdot i+3 \cdot 1 \cdot i^2+i^3$
 $= 1+3i+3i^2+i^3=1+3i+3 \cdot (-1)+i^2 \cdot i$
 $= 1+3i-3-i=-2+2i$
- (6) $(i-3)(i-1)(i+1)(i+3)$
 $= (i-3)(i+3)(i-1)(i+1)$
 $= (i^2-9)(i^2-1)=(-10) \cdot (-2)=20$
- 3 左辺を展開すると、 $x^2+2xyi+y^2i^2=-24i$
 これを整理すると、 $x^2-y^2+2xyi=-24i$
 $x^2-y^2, 2xy$ は実数であるから、
 $x^2-y^2=0 \dots \textcircled{1}$
 $2xy=-24, xy=-12 \dots \textcircled{2}$
 $\textcircled{1}$ より、 $(x+y)(x-y)=0$
 $x=-y$ または $x=y$
 $x=y$ のとき、 $\textcircled{2}$ より、 $y^2=-12$
 この式は成り立たないので適さない。
 $x=-y$ のとき、 $\textcircled{2}$ より、 $y^2=12$
 よって、 $y=2\sqrt{3}, -2\sqrt{3}$
 $y=2\sqrt{3}$ のとき $x=-2\sqrt{3}$
 $y=-2\sqrt{3}$ のとき $x=2\sqrt{3}$
 したがって、 $2\sqrt{3}-2\sqrt{3}i, -2\sqrt{3}+2\sqrt{3}i$

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P.96-97

複素数の割り算をしよう!

- 1 (1) $4-3i$ (2) $-2+5i$
 (3) $2i$ (4) 5
 (5) $\frac{-3i+5}{2}$ (6) $-2i$
- 2 (1) $\frac{-1+i}{2}$ (2) $\frac{3-4i}{5}$ (3) $\frac{1-5i}{2}$
 (4) i (5) $-i$ (6) $1+2i$
 (7) $\frac{-7-i}{2}$ (8) $\frac{-7+24i}{25}$

解き方

- 1 (3) $-2i$ は $0-2i$ と考えます。
 (4) 5 は $5+0i$ と考えます。
 (6) $(1+i)^2=1^2+2i+i^2=1+2i-1=2i$
- 2 (1) $\frac{i}{1-i}=\frac{i(1+i)}{(1-i)(1+i)}=\frac{i+i^2}{1-i^2}=\frac{-1+i}{2}$
- (2) $\frac{5}{3+4i}=\frac{5(3-4i)}{(3+4i)(3-4i)}=\frac{15-20i}{9-16i^2}$
 $=\frac{15-20i}{25}=\frac{3-4i}{5}$
- (3) $\frac{11-16i}{7+3i}=\frac{(11-16i)(7-3i)}{(7+3i)(7-3i)}$
 $=\frac{77-33i-112i+48i^2}{49-9i^2}$
 $=\frac{29-145i}{58}=\frac{1-5i}{2}$
- (4) $\frac{-3+2i}{2+3i}=\frac{(-3+2i)(2-3i)}{(2+3i)(2-3i)}$
 $=\frac{-6+9i+4i-6i^2}{4-9i^2}=\frac{13i}{13}=i$
- (5) $\frac{1-i}{1+i}=\frac{(1-i)^2}{(1+i)(1-i)}=\frac{1-2i+i^2}{1-i^2}$
 $=\frac{-2i}{2}=-i$
- (7) $\frac{(1-2i)^2}{1+i}=\frac{1-4i+4i^2}{1+i}=\frac{-3-4i}{1+i}$
 $=\frac{(-3-4i)(1-i)}{(1+i)(1-i)}=\frac{-3+3i-4i+4i^2}{1-i^2}$
 $=\frac{-7-i}{2}$
- (8) $\frac{2+i}{2-i}=\frac{(2+i)^2}{(2-i)(2+i)}=\frac{4+4i+i^2}{4-i^2}=\frac{3+4i}{5}$
 $\left(\frac{2+i}{2-i}\right)^2=\left(\frac{3+4i}{5}\right)^2=\frac{9+24i+16i^2}{25}$
 $=\frac{-7+24i}{25}$

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P.98-99

確認問題⑦

- 1 (1) 0.6 (2) -5
 (3) 4a (4) -2.1
- 2 (1) $\frac{5}{2} > \sqrt{5}$ (2) $-\sqrt{150} < -12$
- 3 (1) $\sqrt{441}$ (2) $\sqrt{\frac{4}{25}}, -\frac{3}{24}$
 (3) $\frac{4}{33}$ (4) $-\sqrt{8}, \sqrt{\frac{\pi}{3.1415}}, \sqrt{\pi^2}$
- 4 (1) $\frac{56}{99}$ (2) $\frac{452}{333}$
- 5 (1) -5 (2) $-\frac{1}{2}$
- 6 (1) $x = -1, y = 5$ (2) $x = \frac{1}{2}, y = -\frac{7}{2}$
- 7 (1) $\frac{3+3i}{2}$ (2) $\frac{-6+17i}{25}$

解き方

- 1 (1) $\sqrt{0.36} = \sqrt{0.6^2} = 0.6$
- 2 (1) $\left(\frac{5}{2}\right)^2 = \frac{25}{4} = 6.25, (\sqrt{5})^2 = 5,$
 $\left(\frac{5}{2}\right)^2 > (\sqrt{5})^2$ であるから、 $\frac{5}{2} > \sqrt{5}$
- 4 (2) $x = 0.\dot{3}5\dot{7}$ とおく。

$$\begin{array}{r} 1000x = 357.357357357\cdots \\ -) \quad x = 0.357357357\cdots \\ \hline 999x = 357 \end{array}$$
 よって、 $x = \frac{357}{999} = \frac{119}{333}$
 したがって、 $1.\dot{3}5\dot{7} = 1 + \frac{119}{333} = \frac{452}{333}$
- 5 (2) $P = \left|\frac{1}{2} + 2\right| - 2 \left|\frac{1}{2} - 2\right| = \left|\frac{5}{2}\right| - 2 \left|-\frac{3}{2}\right|$
 $= \frac{5}{2} - 2 \cdot \frac{3}{2} = -\frac{1}{2}$
- 6 (2) 左辺を $a+bi$ の形に変形すると、
 $(x-y-4) + (x+y+3)i = 0$
 $x-y-4, x+y+3$ は実数であるから、
 $x-y-4=0 \cdots \textcircled{1}, x+y+3=0 \cdots \textcircled{2}$
 $\textcircled{1}, \textcircled{2}$ より、 $x = \frac{1}{2}, y = -\frac{7}{2}$
- 7 (1) $\frac{3i}{1+i} = \frac{3i(1-i)}{(1+i)(1-i)} = \frac{3i-3i^2}{1-i^2}$
 $= \frac{3i-3 \cdot (-1)}{1+1} = \frac{3+3i}{2}$

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P.100-101

根号を含む式を変形しよう!

- 1 (1) $2\sqrt{10}$ (2) $\frac{\sqrt{3}}{7}$
 (3) $10\sqrt{5}$ (4) $\frac{\sqrt{3}}{10}$
- 2 (1) $6\sqrt{5}$ (2) $15\sqrt{3}$
 (3) $14\sqrt{2}$ (4) $6\sqrt{21}$
- 3 (1) $\sqrt{48}$ (2) $\sqrt{192}$
 (3) $\sqrt{5}$ (4) $\sqrt{\frac{2}{5}}$
- 4 (1) $x=3$ (2) $x=15$
 (3) $x=15$ (4) $x=1$

解き方

- 1 (1) $\sqrt{40} = \sqrt{2^2 \cdot 10} = 2\sqrt{10}$
 (2) $\sqrt{\frac{3}{49}} = \sqrt{\frac{3}{7^2}} = \frac{\sqrt{3}}{7}$
 (3) $\sqrt{500} = \sqrt{10^2 \cdot 5} = 10\sqrt{5}$
 (4) $\sqrt{0.03} = \sqrt{\frac{3}{100}} = \sqrt{\frac{3}{10^2}} = \frac{\sqrt{3}}{10}$
- 2 (1) 根号の中の数を素因数分解して、

$$\begin{array}{r} 2) 180 \\ 2) 90 \\ 3) 45 \\ 3) 15 \\ \hline 5 \end{array}$$
 $\sqrt{180} = \sqrt{2^2 \cdot 3^2 \cdot 5}$
 $= 2 \cdot 3 \sqrt{5}$
 $= 6\sqrt{5}$
- (2) 根号の中の数を素因数分解して、
 $\sqrt{675} = \sqrt{3^2 \cdot 5^2 \cdot 3} = 3 \cdot 5 \sqrt{3} = 15\sqrt{3}$
- 3 (1) $4\sqrt{3} = \sqrt{4^2 \cdot 3} = \sqrt{48}$
 (3) $\frac{\sqrt{20}}{2} = \frac{\sqrt{20}}{\sqrt{2^2}} = \frac{\sqrt{20}}{\sqrt{4}} = \sqrt{\frac{20}{4}} = \sqrt{5}$
- 4 (1) $\sqrt{48x} = \sqrt{4^2 \cdot 3x}$ より、与式の値が整数となる最小の正の整数 x は、 $x=3$
- (3) $\sqrt{\frac{6615}{x}} = \sqrt{\frac{3^2 \cdot 7^2 \cdot 3 \cdot 5}{x}}$ より、与式の値が整数となる最小の正の整数 x は、
 $x=3 \cdot 5 = 15$

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P.102-103

根号を含む式の掛け算と割り算をしよう!

- 1 (1) $\sqrt{15}$ (2) 6
 (3) $\sqrt{5}$ (4) 2
- 2 (1) $5\sqrt{2}$ (2) $-4\sqrt{15}$
 (3) $50\sqrt{3}$ (4) $18\sqrt{6}$
 (5) 8 (6) $\sqrt{2}$
 (7) $9\sqrt{2}$ (8) 2
- 3 (1) $\sqrt{30}$ (2) $3\sqrt{3}$
 (3) -5 (4) 3
 (5) $2\sqrt{6}$ (6) $\frac{2\sqrt{3}}{3}$
 (7) 27 (8) $\frac{3}{2}$

解き方

- 1 (1) $\sqrt{5} \times \sqrt{3} = \sqrt{5 \cdot 3} = \sqrt{15}$
 (3) $\frac{\sqrt{10}}{\sqrt{2}} = \sqrt{\frac{10}{2}} = \sqrt{5}$
- 2 (1) $\sqrt{10} \times \sqrt{5} = \sqrt{5 \cdot 2} \times \sqrt{5} = \sqrt{5^2 \cdot 2} = 5\sqrt{2}$
 (3) $2\sqrt{15} \times \sqrt{125} = 2\sqrt{3 \cdot 5} \times \sqrt{5^2 \cdot 5}$
 $= 2\sqrt{3 \cdot 5^4} = 2 \cdot 5^2 \sqrt{3} = 50\sqrt{3}$
 (5) $\sqrt{192} \div \sqrt{3} = \sqrt{\frac{192}{3}} = \sqrt{64} = \sqrt{8^2} = 8$
 (6) $4\sqrt{24} \div 8\sqrt{3} = 4\sqrt{2^3 \cdot 3} \div 8\sqrt{3}$
 $= 4 \cdot 2\sqrt{2 \cdot 3} \div 8\sqrt{3} = \frac{4 \cdot 2\sqrt{2 \cdot 3}}{8\sqrt{3}} = \sqrt{2}$
- 3 (1) $\sqrt{2} \times \sqrt{3} \times \sqrt{5} = \sqrt{2 \cdot 3 \cdot 5} = \sqrt{30}$
 (2) $\sqrt{180} \div 2\sqrt{5} \times \sqrt{3} = \frac{6\sqrt{5} \times \sqrt{3}}{2\sqrt{5}}$
 $= 3 \times \sqrt{3} = 3\sqrt{3}$
 (3) $(-2\sqrt{15}) \times \sqrt{5} \div 2\sqrt{3}$
 $= (-2\sqrt{15}) \times \frac{\sqrt{5}}{2\sqrt{3}}$
 $= -\sqrt{5} \times \sqrt{5} = -5$
 (6) $\sqrt{8} \times \sqrt{6} \div \sqrt{12} \div \sqrt{3} = \frac{\sqrt{8} \times \sqrt{6}}{\sqrt{12} \times \sqrt{3}}$
 $= \frac{2\sqrt{2} \times \sqrt{6}}{2\sqrt{3} \times \sqrt{3}} = \frac{2 \times 2 \times \sqrt{3}}{2 \times 3} = \frac{2\sqrt{3}}{3}$
 (7) $3\sqrt{24} \div 2\sqrt{2} \times \sqrt{27} = \frac{3\sqrt{24} \times \sqrt{27}}{2\sqrt{2}}$
 $= \frac{3 \cdot 2\sqrt{6} \times 3\sqrt{3}}{2\sqrt{2}}$
 $= 3\sqrt{3} \times 3\sqrt{3} = 27$

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P.104-105

根号を含む式の足し算と引き算をしよう!

- 1 (1) $8\sqrt{2}$ (2) $4\sqrt{3}$
 (3) $5\sqrt{3}$ (4) $2\sqrt{2}$
 (5) $19\sqrt{2}$ (6) 0
 (7) $13\sqrt{5}$ (8) $9\sqrt{3}$
 (9) $5\sqrt{2}$ (10) $-\sqrt{6}$
- 2 (1) $2\sqrt{3} + 3\sqrt{2}$ (2) $\sqrt{3} - 3\sqrt{2}$
 (3) $2\sqrt{15} - 3\sqrt{5}$ (4) $3\sqrt{2} - 2\sqrt{3}$
 (5) $\sqrt{6} - \sqrt{2} + 2\sqrt{3}$
 (6) $-3\sqrt{7} + 7\sqrt{2} - 7\sqrt{3}$
 (7) $\frac{25-4\sqrt{2}}{2}$ (8) $-2 + 2\sqrt{2}$

解き方

- 1 (3) $\sqrt{12} + \sqrt{27} = \sqrt{2^2 \cdot 3} + \sqrt{3^2 \cdot 3}$
 $= 2\sqrt{3} + 3\sqrt{3} = (2+3)\sqrt{3} = 5\sqrt{3}$
 (4) $\sqrt{98} - \sqrt{50} = \sqrt{7^2 \cdot 2} - \sqrt{5^2 \cdot 2}$
 $= 7\sqrt{2} - 5\sqrt{2} = (7-5)\sqrt{2} = 2\sqrt{2}$
 (5) $3\sqrt{18} + 2\sqrt{50} = 3\sqrt{3^2 \cdot 2} + 2\sqrt{5^2 \cdot 2}$
 $= 3 \cdot 3\sqrt{2} + 2 \cdot 5\sqrt{2} = 9\sqrt{2} + 10\sqrt{2} = 19\sqrt{2}$
 (7) $6\sqrt{20} + \sqrt{45} - 2\sqrt{5}$
 $= 6\sqrt{2^2 \cdot 5} + \sqrt{3^2 \cdot 5} - 2\sqrt{5}$
 $= 6 \cdot 2\sqrt{5} + 3\sqrt{5} - 2\sqrt{5}$
 $= 12\sqrt{5} + 3\sqrt{5} - 2\sqrt{5} = 13\sqrt{5}$
 (9) $-\sqrt{32} + 4\sqrt{2} + \sqrt{50}$
 $= -\sqrt{4^2 \cdot 2} + 4\sqrt{2} + \sqrt{5^2 \cdot 2}$
 $= -4\sqrt{2} + 4\sqrt{2} + 5\sqrt{2} = 5\sqrt{2}$
- 2 (1) $\sqrt{2}(\sqrt{6} + 3) = \sqrt{2} \times \sqrt{6} + \sqrt{2} \times 3$
 $= \sqrt{12} + 3\sqrt{2} = 2\sqrt{3} + 3\sqrt{2}$
 (3) $\sqrt{5}(2\sqrt{3} - 3) = \sqrt{5} \times 2\sqrt{3} - \sqrt{5} \times 3$
 $= 2\sqrt{15} - 3\sqrt{5}$
 (4) $\sqrt{6}(\sqrt{3} - \sqrt{2}) = \sqrt{6} \times \sqrt{3} - \sqrt{6} \times \sqrt{2}$
 $= \sqrt{18} - \sqrt{12} = 3\sqrt{2} - 2\sqrt{3}$
 (5) $\sqrt{2}(\sqrt{3} - 1 + \sqrt{6})$
 $= \sqrt{2} \times \sqrt{3} - \sqrt{2} \times 1 + \sqrt{2} \times \sqrt{6}$
 $= \sqrt{6} - \sqrt{2} + \sqrt{12} = \sqrt{6} - \sqrt{2} + 2\sqrt{3}$
 (8) $(\sqrt{20} + \sqrt{40} - \sqrt{80}) \div \sqrt{5}$
 $= (2\sqrt{5} + 2\sqrt{10} - 4\sqrt{5}) \times \frac{1}{\sqrt{5}}$
 $= 2\sqrt{5} \times \frac{1}{\sqrt{5}} + 2\sqrt{10} \times \frac{1}{\sqrt{5}} - 4\sqrt{5} \times \frac{1}{\sqrt{5}}$
 $= 2 + 2\sqrt{2} - 4 = -2 + 2\sqrt{2}$

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P.106-107

展開の公式を使って計算しよう!

- 1 (1) $7+4\sqrt{3}$ (2) $32-10\sqrt{7}$
 (3) $13-4\sqrt{3}$ (4) $15+4\sqrt{14}$
 (5) $8-4\sqrt{3}$ (6) 11
 (7) 3 (8) 2

- 2 (1) $\sqrt{6}+2\sqrt{2}+2\sqrt{3}+4$
 (2) $3\sqrt{2}-\sqrt{6}+2\sqrt{3}-2$
 (3) $-2-2\sqrt{6}$ (4) $10-7\sqrt{2}$
 (5) $\sqrt{5}$
 (6) $6\sqrt{2}+2\sqrt{3}+6\sqrt{6}+6$
 (7) $21-11\sqrt{6}$ (8) $6-9\sqrt{6}$
 (9) $4+2\sqrt{6}-\sqrt{3}$ (10) $3+2\sqrt{10}+\sqrt{2}$

解き方

- 1 (1) $(2+\sqrt{3})^2=2^2+2\cdot 2\cdot\sqrt{3}+(\sqrt{3})^2$
 $=4+4\sqrt{3}+3=7+4\sqrt{3}$
 (3) $(2\sqrt{3}-1)^2=(2\sqrt{3})^2-2\cdot 2\sqrt{3}\cdot 1+1^2$
 $=12-4\sqrt{3}+1=13-4\sqrt{3}$
 (6) $(4+\sqrt{5})(4-\sqrt{5})$
 $=4^2-(\sqrt{5})^2=16-5=11$
 (8) $(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})$
 $=(\sqrt{5})^2-(\sqrt{3})^2=5-3=2$
 2 (3) $(\sqrt{6}+2)(\sqrt{6}-4)$
 $=(\sqrt{6})^2+(2-4)\sqrt{6}-8$
 $=6-2\sqrt{6}-8=-2-2\sqrt{6}$
 (4) $(\sqrt{2}-2)(2\sqrt{2}-3)$
 $=2(\sqrt{2})^2+(-3-4)\sqrt{2}+6$
 $=4-7\sqrt{2}+6=10-7\sqrt{2}$
 (5) $(\sqrt{15}-\sqrt{10})(\sqrt{3}+\sqrt{2})$
 $=\sqrt{3^2\cdot 5}+\sqrt{30}-\sqrt{30}-\sqrt{2^2\cdot 5}$
 $=3\sqrt{5}+\sqrt{30}-\sqrt{30}-2\sqrt{5}=\sqrt{5}$
 (7) $(\sqrt{3}-3\sqrt{2})(\sqrt{27}-\sqrt{8})$
 $=(\sqrt{3}-3\sqrt{2})(3\sqrt{3}-2\sqrt{2})$
 $=3(\sqrt{3})^2+(-2-9)\sqrt{3}\sqrt{2}+6(\sqrt{2})^2$
 $=9-11\sqrt{6}+12=21-11\sqrt{6}$
 (8) $(3\sqrt{2}+\sqrt{3})(3\sqrt{2}-4\sqrt{3})$
 $=(3\sqrt{2})^2+(-12+3)\sqrt{2}\sqrt{3}-4(\sqrt{3})^2$
 $=18-9\sqrt{6}-12=6-9\sqrt{6}$
 (9) $(\sqrt{3}+\sqrt{2})^2-(\sqrt{3}-1)(\sqrt{3}+2)$
 $=(3+2\sqrt{6}+2)-(3+2\sqrt{3}-\sqrt{3}-2)$
 $=3+2\sqrt{6}+2-3-2\sqrt{3}+\sqrt{3}+2$
 $=4+2\sqrt{6}-\sqrt{3}$

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P.108-109

負の数の平方根の計算をしよう!

- 1 (1) $\pm\sqrt{5}i$ (2) $\pm\sqrt{17}i$
 2 (1) $\sqrt{6}i$ (2) $3i$
 (3) $2\sqrt{3}i$ (4) $5\sqrt{2}i$
 3 (1) $3i$ (2) $\sqrt{3}-2+7i$
 (3) $-4\sqrt{6}$ (4) -48
 (5) $-\sqrt{3}i$ (6) 3
 (7) $11+2\sqrt{3}i$ (8) $27-38i$

解き方

- 1 (1) $\pm\sqrt{-5}=\pm\sqrt{5}i$
 (2) $\pm\sqrt{-17}=\pm\sqrt{17}i$
 2 $a>0$ のとき、 $\sqrt{-a}=\sqrt{a}i$ であることを使
 って求めるよ。
 (2) $\sqrt{-9}=\sqrt{9}i=3i$
 (3) $\sqrt{-12}=\sqrt{12}i=2\sqrt{3}i$
 (4) $\sqrt{-50}=\sqrt{50}i=5\sqrt{2}i$
 3 (1) $\sqrt{-49}-\sqrt{-16}=\sqrt{49}i-\sqrt{16}i$
 $=7i-4i=3i$
 (2) $(\sqrt{-4}+\sqrt{3})-(2-\sqrt{-25})$
 $=(\sqrt{4}i+\sqrt{3})-(2-\sqrt{25}i)$
 $=(2i+\sqrt{3})-(2-5i)=2i+\sqrt{3}-2+5i$
 $=\sqrt{3}-2+7i$
 (3) $\sqrt{-12}\times\sqrt{-8}=\sqrt{12}i\times\sqrt{8}i$
 $=2\sqrt{3}i\times 2\sqrt{2}i=4\sqrt{6}i^2=-4\sqrt{6}$
 (4) $\sqrt{-72}\times\sqrt{-32}=\sqrt{72}i\times\sqrt{32}i$
 $=6\sqrt{2}i\times 4\sqrt{2}i=24(\sqrt{2})^2i^2=-48$
 (5) $\frac{\sqrt{48}}{\sqrt{-16}}=\frac{\sqrt{48}}{\sqrt{16}i}=\frac{4\sqrt{3}}{4i}=\frac{\sqrt{3}}{i}=\frac{\sqrt{3}i}{i^2}$
 $=\frac{\sqrt{3}i}{-1}=-\sqrt{3}i$
 (6) $\frac{\sqrt{-27}}{\sqrt{-3}}=\frac{\sqrt{27}i}{\sqrt{3}i}=\frac{3\sqrt{3}i}{\sqrt{3}i}=3$
 (7) $(\sqrt{-3}+2)(4-\sqrt{-3})$
 $=(\sqrt{3}i+2)(4-\sqrt{3}i)$
 $=4\sqrt{3}i-3i^2+8-2\sqrt{3}i$
 $=4\sqrt{3}i+3+8-2\sqrt{3}i=11+2\sqrt{3}i$
 (8) $(7-\sqrt{-4})(5-\sqrt{-16})$
 $=(7-\sqrt{4}i)(5-\sqrt{16}i)$
 $=(7-2i)(5-4i)=35-28i-10i+8i^2$
 $=35-28i-10i-8=27-38i$

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P.110-111

確認問題⑧

- 1 (1) $6\sqrt{2}$ (2) $4\sqrt{3}$
 (3) $14\sqrt{5}$ (4) $35\sqrt{3}$
- 2 (1) $6\sqrt{3}$ (2) $\frac{\sqrt{2}}{7}$
 (3) $280\sqrt{3}$ (4) 6
- 3 (1) $4\sqrt{7}$ (2) $17\sqrt{2}$
 (3) $15-5\sqrt{2}$ (4) 4
- 4 (1) $5+2\sqrt{6}$ (2) $7-4\sqrt{3}$
 (3) 14 (4) $60+6\sqrt{7}$
- 5 (1) $\frac{\sqrt{2}}{3}$ (2) $7+5\sqrt{2}i$

解き方

- 1 (1) $\sqrt{72} = \sqrt{6^2 \cdot 2} = 6\sqrt{2}$
 (3) $\sqrt{980} = \sqrt{2^2 \cdot 7^2 \cdot 5} = 2 \cdot 7\sqrt{5} = 14\sqrt{5}$
- 2 (1) $3\sqrt{2} \times \sqrt{6} = 3\sqrt{2} \times \sqrt{2 \cdot 3} = 3\sqrt{2^2 \cdot 3}$
 $= 3 \cdot 2\sqrt{3} = 6\sqrt{3}$
 (2) $\frac{\sqrt{6}}{\sqrt{7}} \div \sqrt{21} = \frac{\sqrt{6}}{\sqrt{7}} \times \frac{1}{\sqrt{21}}$
 $= \frac{\sqrt{6}}{\sqrt{7}} \times \frac{1}{\sqrt{7 \cdot 3}} = \frac{\sqrt{6}}{7\sqrt{3}} = \frac{\sqrt{3 \cdot 2}}{7\sqrt{3}} = \frac{\sqrt{2}}{7}$
- 3 (1) $3\sqrt{28} - 2\sqrt{7} = 3\sqrt{2^2 \cdot 7} - 2\sqrt{7}$
 $= 3 \cdot 2\sqrt{7} - 2\sqrt{7} = 6\sqrt{7} - 2\sqrt{7} = 4\sqrt{7}$
 (2) $3\sqrt{2} + \sqrt{72} + 2\sqrt{32}$
 $= 3\sqrt{2} + \sqrt{6^2 \cdot 2} + 2\sqrt{4^2 \cdot 2}$
 $= 3\sqrt{2} + 6\sqrt{2} + 8\sqrt{2} = 17\sqrt{2}$
 (4) $(2\sqrt{18} - \sqrt{8}) \div \sqrt{2}$
 $= (2\sqrt{18} - \sqrt{8}) \times \frac{1}{\sqrt{2}} = 2\sqrt{9} - \sqrt{4}$
 $= 2 \cdot 3 - 2 = 4$
- 4 (1) $(\sqrt{2} + \sqrt{3})^2$
 $= (\sqrt{2})^2 + 2 \cdot \sqrt{2} \cdot \sqrt{3} + (\sqrt{3})^2$
 $= 2 + 2\sqrt{6} + 3 = 5 + 2\sqrt{6}$
 (3) $(2\sqrt{5} + \sqrt{6})(2\sqrt{5} - \sqrt{6})$
 $= (2\sqrt{5})^2 - (\sqrt{6})^2$
 $= 20 - 6 = 14$
- 5 (2) $(3 - \sqrt{-2})(1 + \sqrt{-8})$
 $= (3 - \sqrt{2}i)(1 + 2\sqrt{2}i)$
 $= 3 + 6\sqrt{2}i - \sqrt{2}i - 2(\sqrt{2})^2 i^2$
 $= 3 + 6\sqrt{2}i - \sqrt{2}i - 4i^2$
 $= 3 + 6\sqrt{2}i - \sqrt{2}i + 4$
 $= 7 + 5\sqrt{2}i$

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P.112-113

有理化って何?

- 1 (1) $\frac{\sqrt{5}}{5}$ (2) $\frac{2\sqrt{3}}{3}$ (3) $\frac{3\sqrt{5}}{2}$
 (4) $\frac{\sqrt{14}}{2}$ (5) $\frac{\sqrt{3}}{2}$ (6) $2 - 2\sqrt{3}$
- 2 (1) $\sqrt{7} - \sqrt{5}$ (2) $2 + \sqrt{3}$
 (3) $4 - \sqrt{15}$ (4) $5 + 2\sqrt{6}$
- 3 (1) 0.707 (2) 7.464

解き方

- 1 (1) $\frac{1}{\sqrt{5}} = \frac{1 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{\sqrt{5}}{5}$
 (2) $\frac{2}{\sqrt{3}} = \frac{2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{2\sqrt{3}}{3}$
 (3) $\frac{15}{\sqrt{20}} = \frac{15}{2\sqrt{5}} = \frac{15 \times \sqrt{5}}{2\sqrt{5} \times \sqrt{5}} = \frac{15 \times \sqrt{5}}{2 \times 5}$
 $= \frac{3\sqrt{5}}{2}$
 (4) $\frac{\sqrt{21}}{\sqrt{6}} = \frac{\sqrt{21} \div \sqrt{3}}{\sqrt{6} \div \sqrt{3}} = \frac{\sqrt{7}}{\sqrt{2}} = \frac{\sqrt{7} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$
 $= \frac{\sqrt{14}}{2}$
 (5) $\frac{\sqrt{24}}{\sqrt{32}} = \frac{\sqrt{24} \div \sqrt{8}}{\sqrt{32} \div \sqrt{8}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$
 (6) $\frac{\sqrt{12} - 6}{\sqrt{3}} = \frac{2\sqrt{3} - 6}{\sqrt{3}} = \frac{(2\sqrt{3} - 6) \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$
 $= \frac{2(\sqrt{3})^2 - 6\sqrt{3}}{3} = \frac{6 - 6\sqrt{3}}{3}$
 $= 2 - 2\sqrt{3}$
- 2 (3) $\frac{\sqrt{5} - \sqrt{3}}{\sqrt{5} + \sqrt{3}} = \frac{(\sqrt{5} - \sqrt{3})^2}{(\sqrt{5} + \sqrt{3})(\sqrt{5} - \sqrt{3})}$
 $= \frac{5 - 2\sqrt{15} + 3}{5 - 3} = \frac{8 - 2\sqrt{15}}{2} = 4 - \sqrt{15}$
- (4) $\frac{3\sqrt{2} + 2\sqrt{3}}{3\sqrt{2} - 2\sqrt{3}}$
 $= \frac{(3\sqrt{2} + 2\sqrt{3})^2}{(3\sqrt{2} - 2\sqrt{3})(3\sqrt{2} + 2\sqrt{3})}$
 $= \frac{18 + 12\sqrt{6} + 12}{18 - 12} = \frac{30 + 12\sqrt{6}}{6}$
 $= 5 + 2\sqrt{6}$
- 3 (1) $\frac{1}{\sqrt{2}} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{2} = \frac{1.414}{2}$
 $= 0.707$
 (2) $\frac{2}{2 - \sqrt{3}} = \frac{2 \times (2 + \sqrt{3})}{(2 - \sqrt{3})(2 + \sqrt{3})}$
 $= \frac{4 + 2\sqrt{3}}{2^2 - (\sqrt{3})^2} = \frac{4 + 2\sqrt{3}}{4 - 3} = 4 + 2\sqrt{3}$
 $= 4 + 2 \times 1.732 = 7.464$

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P.114-115

分母を有理化して足し算と引き算をしよう!

- 1 (1) $\frac{\sqrt{2}}{2} + \frac{2\sqrt{5}}{5}$ (2) $-\frac{\sqrt{6}}{2}$
 (3) $\frac{\sqrt{3}}{3}$ (4) $\frac{\sqrt{5}}{10} - \frac{\sqrt{3}}{9}$
 (5) $2 + \frac{\sqrt{3}}{3} - \frac{\sqrt{7}}{7}$ (6) $\frac{43\sqrt{3}}{5}$
- 2 (1) -4 (2) $\frac{\sqrt{2}}{5}$
 (3) $9 + 2\sqrt{6} - \sqrt{15}$ (4) $\frac{7 + \sqrt{3}}{2}$

解き方

- 1 (1) $\frac{1}{\sqrt{2}} + \frac{2}{\sqrt{5}} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} + \frac{2 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}}$
 $= \frac{\sqrt{2}}{2} + \frac{2\sqrt{5}}{5}$
- (2) $\frac{\sqrt{3}}{2} - \frac{6}{\sqrt{6}} = \frac{\sqrt{3}}{2} - \frac{6}{\sqrt{2} \cdot \sqrt{3}} = \frac{\sqrt{3} \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} - \frac{6 \times \sqrt{6}}{\sqrt{6} \times \sqrt{6}} = \frac{\sqrt{6}}{2} - \frac{6\sqrt{6}}{6}$
 $= \frac{\sqrt{6}}{2} - \sqrt{6} = -\frac{\sqrt{6}}{2}$
- (5) $\frac{2\sqrt{3}+1}{\sqrt{3}} - \frac{1}{\sqrt{7}} = 2 + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{7}}$
 $= 2 + \frac{\sqrt{3}}{3} - \frac{\sqrt{7}}{7}$
- 2 (1) $\frac{2}{\sqrt{6}+2} - (\sqrt{6}+2)$
 $= \frac{2(\sqrt{6}-2)}{(\sqrt{6}+2)(\sqrt{6}-2)} - (\sqrt{6}+2)$
 $= \frac{2\sqrt{6}-4}{(\sqrt{6})^2-2^2} - (\sqrt{6}+2)$
 $= \frac{2\sqrt{6}-4}{6-4} - (\sqrt{6}+2)$
 $= \sqrt{6}-2-\sqrt{6}-2=-4$
- (3) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$
 $= \frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})} + \frac{(\sqrt{5}-\sqrt{3})^2}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})}$
 $= \frac{(\sqrt{3}+\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2} + \frac{(\sqrt{5}-\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2}$
 $= 3+2\sqrt{6}+2 + \frac{5-2\sqrt{15}+3}{5-3}$
 $= 5+2\sqrt{6}+4-\sqrt{15}=9+2\sqrt{6}-\sqrt{15}$

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P.116-117

式の値を求めよう!①

- 1 (1) $3-4\sqrt{3}$ (2) $24-8\sqrt{6}$
 (3) 5 (4) 3
 (5) $\frac{5-\sqrt{5}}{2}$
- 2 (1) ① $8\sqrt{3}$ ② $4\sqrt{3}+6$
 (2) ① 8 ② $12-4\sqrt{3}$
 (3) ① 6 ② $10\sqrt{2}$

解き方

1 (1) 因数分解の利用

$$\begin{aligned} \text{与式} &= (x+1)(x-3) \\ &= \{(\sqrt{3}-1)+1\}\{(\sqrt{3}-1)-3\} \\ &= \sqrt{3} \times (\sqrt{3}-4) = 3-4\sqrt{3} \end{aligned}$$

(3) 因数分解の利用

$$\begin{aligned} \text{与式} &= (x+2)^2 = \{(\sqrt{5}-2)+2\}^2 = (\sqrt{5})^2 \\ &= 5 \end{aligned}$$

(5) 因数分解の利用

$$\begin{aligned} a &= \frac{1+\sqrt{5}}{2} \text{ より、} \\ 2a &= 1+\sqrt{5} \quad 2a-1=\sqrt{5} \\ \text{与式} &= 2a^2-3a+1 \\ &= (2a-1)(a-1) = \sqrt{5} \left(\frac{1+\sqrt{5}}{2} - 1 \right) \\ &= \sqrt{5} \times \frac{\sqrt{5}-1}{2} = \frac{5-\sqrt{5}}{2} \end{aligned}$$

2 (1) ① $x^2-y^2=(x+y)(x-y)$

$$\begin{aligned} &= \{(2+\sqrt{3})+(2-\sqrt{3})\} \\ &\quad \times \{(2+\sqrt{3})-(2-\sqrt{3})\} \\ &= 4 \cdot 2\sqrt{3} = 8\sqrt{3} \end{aligned}$$

② $x^2-xy=x(x-y)$

$$\begin{aligned} &= (2+\sqrt{3})\{(2+\sqrt{3})-(2-\sqrt{3})\} \\ &= (2+\sqrt{3}) \cdot 2\sqrt{3} = 4\sqrt{3}+6 \end{aligned}$$

(3) ① $x^2+y^2=(x+y)^2-2xy$

$$\begin{aligned} &= \{(\sqrt{2}+1)+(\sqrt{2}-1)\}^2 \\ &\quad - 2(\sqrt{2}+1)(\sqrt{2}-1) \\ &= (2\sqrt{2})^2 - 2\{(\sqrt{2})^2 - 1^2\} \\ &= 8 - 2(2-1) = 6 \end{aligned}$$

② $x^3+y^3=(x+y)^3-3xy(x+y)$

$$\begin{aligned} &= (2\sqrt{2})^3 - 3(\sqrt{2}+1)(\sqrt{2}-1) \cdot 2\sqrt{2} \\ &= 16\sqrt{2} - 6\sqrt{2} = 10\sqrt{2} \end{aligned}$$

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P.118-119

式の値を求めよう!②

$$\begin{array}{ll} \text{1} & (1) 2\sqrt{5} \quad (2) 2 \\ & (3) 16 \quad (4) 28\sqrt{5} \end{array}$$

$$\begin{array}{ll} \text{2} & (1) \sqrt{6} \quad (2) \frac{1}{2} \\ & (3) 5 \quad (4) \frac{\sqrt{6}}{2} \end{array}$$

$$\text{3} \quad (1) -5 \quad (2) 25$$

解き方

$$\begin{aligned} \text{1} \quad (1) \quad x+y &= \frac{2}{\sqrt{5}+\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} \\ &= \frac{2(\sqrt{5}-\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} \\ &\quad + \frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})} \\ &= \frac{2(\sqrt{5}-\sqrt{3})+2(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} \\ &= \frac{4\sqrt{5}}{5-3} = 2\sqrt{5} \end{aligned}$$

$$\begin{aligned} (2) \quad xy &= \frac{2}{\sqrt{5}+\sqrt{3}} \cdot \frac{2}{\sqrt{5}-\sqrt{3}} \\ &= \frac{4}{(\sqrt{5}+\sqrt{3})(\sqrt{5}-\sqrt{3})} = 2 \end{aligned}$$

$$\begin{aligned} (3) \quad x^2+y^2 &= (x+y)^2 - 2xy \\ &= (2\sqrt{5})^2 - 2 \cdot 2 = 16 \end{aligned}$$

$$\begin{aligned} (4) \quad x^3+y^3 &= (x+y)^3 - 3xy(x+y) \\ &= (2\sqrt{5})^3 - 3 \cdot 2 \cdot 2\sqrt{5} = 28\sqrt{5} \end{aligned}$$

$$\begin{aligned} \text{2} \quad (1) \quad x+y &= \frac{1}{\sqrt{6}+2} + \frac{1}{\sqrt{6}-2} \\ &= \frac{\sqrt{6}-2+\sqrt{6}+2}{(\sqrt{6}+2)(\sqrt{6}-2)} = \frac{2\sqrt{6}}{6-4} = \sqrt{6} \end{aligned}$$

$$\begin{aligned} (2) \quad xy &= \frac{1}{\sqrt{6}+2} \cdot \frac{1}{\sqrt{6}-2} \\ &= \frac{1}{(\sqrt{6}+2)(\sqrt{6}-2)} = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} (3) \quad x^2+y^2 &= (x+y)^2 - 2xy \\ &= (\sqrt{6})^2 - 2 \cdot \frac{1}{2} = 5 \end{aligned}$$

$$(4) \quad x^2y+xy^2 = xy(x+y) = \frac{1}{2} \cdot \sqrt{6} = \frac{\sqrt{6}}{2}$$

$$\begin{aligned} \text{3} \quad (1) \quad 2x-3y &= 2 \cdot \frac{\sqrt{3}-\sqrt{2}}{\sqrt{2}} - 3 \cdot \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}} \\ &= \sqrt{2}(\sqrt{3}-\sqrt{2}) - \sqrt{3}(\sqrt{3}+\sqrt{2}) \\ &= \sqrt{6} - 2 - 3 - \sqrt{6} = -5 \end{aligned}$$

$$\begin{aligned} (2) \quad 4x^2-12xy+9y^2 &= (2x-3y)^2 \\ &= (-5)^2 = 25 \end{aligned}$$

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P.120-121

確認問題⑨

$$\begin{array}{ll} \text{1} & (1) \frac{\sqrt{6}}{3} \quad (2) 3-\sqrt{3} \\ & (3) -4+\sqrt{15} \quad (4) 3+\sqrt{10} \end{array}$$

$$\text{2} \quad (1) \frac{2\sqrt{5}}{5} - \frac{3\sqrt{2}}{10} \quad (2) \frac{\sqrt{6}}{2}$$

$$(3) -1+2\sqrt{2}-\sqrt{3} \quad (4) \frac{26}{11}$$

$$\text{3} \quad (1) 4\sqrt{21} \quad (2) 8\sqrt{7}$$

$$\text{4} \quad (1) 8 \quad (2) \frac{9-3\sqrt{7}}{2}$$

解き方

$$\begin{aligned} \text{1} \quad (2) \quad \frac{\sqrt{12}}{\sqrt{3}+1} &= \frac{2\sqrt{3}(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)} \\ &= \frac{6-2\sqrt{3}}{3-1} = 3-\sqrt{3} \end{aligned}$$

$$\begin{aligned} (3) \quad \frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}+\sqrt{5}} &= \frac{(\sqrt{3}-\sqrt{5})^2}{(\sqrt{3}+\sqrt{5})(\sqrt{3}-\sqrt{5})} \\ &= \frac{3-2\sqrt{15}+5}{3-5} = \frac{8-2\sqrt{15}}{-2} = -4+\sqrt{15} \end{aligned}$$

$$\begin{aligned} \text{2} \quad (3) \quad \frac{1}{1+\sqrt{2}} - \frac{1}{\sqrt{2}+\sqrt{3}} &= \frac{1}{\sqrt{2}+1} - \frac{1}{\sqrt{3}+\sqrt{2}} \\ &= \frac{\sqrt{2}-1}{2-1} - \frac{\sqrt{3}-\sqrt{2}}{3-2} \\ &= \sqrt{2}-1 - (\sqrt{3}-\sqrt{2}) = -1+2\sqrt{2}-\sqrt{3} \end{aligned}$$

$$\begin{aligned} \text{3} \quad (1) \quad a^2-b^2 &= (a+b)(a-b) \\ &= \{(\sqrt{7}+\sqrt{3})+(\sqrt{7}-\sqrt{3})\} \\ &\quad \times \{(\sqrt{7}+\sqrt{3})-(\sqrt{7}-\sqrt{3})\} \\ &= 2\sqrt{7} \cdot 2\sqrt{3} = 4\sqrt{21} \end{aligned}$$

$$\begin{aligned} (2) \quad a^2b+ab^2 &= ab(a+b) \\ &= (\sqrt{7}+\sqrt{3})(\sqrt{7}-\sqrt{3}) \\ &\quad \times \{(\sqrt{7}+\sqrt{3})+(\sqrt{7}-\sqrt{3})\} \\ &= (7-3) \cdot 2\sqrt{7} = 8\sqrt{7} \end{aligned}$$

$$\begin{aligned} \text{4} \quad x+y &= \frac{1}{3+\sqrt{7}} + \frac{1}{3-\sqrt{7}} \\ &= \frac{3-\sqrt{7}}{9-7} + \frac{3+\sqrt{7}}{9-7} = 3 \end{aligned}$$

$$\begin{aligned} xy &= \frac{1}{3+\sqrt{7}} \cdot \frac{1}{3-\sqrt{7}} \\ &= \frac{1}{(3+\sqrt{7})(3-\sqrt{7})} = \frac{1}{2} \end{aligned}$$

$$(1) \quad x^2+y^2 = (x+y)^2 - 2xy = 3^2 - 2 \cdot \frac{1}{2} = 8$$

$$(2) \quad x^2+xy = x(x+y) = \frac{3-\sqrt{7}}{2} \cdot 3 = \frac{9-3\sqrt{7}}{2}$$

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P.122-123

平方根と複素数のまとめ①

- 1 (1) $\pm\sqrt{11}$ (2) $\pm 2\sqrt{3}i$
- 2 (1) $\frac{1}{2}$ (2) 3
- 3 $a=4, 5, 6, 7$
- 4 (1) $0.\dot{6}2\dot{9}$ (2) $0.\dot{0}\dot{6}$
- 5 (1) $\frac{14}{9}$ (2) $\frac{311}{99}$
- 6 (1) $21-13i$ (2) $-\frac{2+11i}{5}$
 (3) $-3-4i$ (4) $\frac{36+2i}{25}$
- 7 (1) -5 と 5
 (2) $-4, -3, -2, -1, 0, 1, 2, 3, 4$
- 8 (1) -10 (2) $-2\pi+8$

解き方

- 2 (2) $\sqrt{(-3)^2}=\sqrt{3^2}=3$
- 3 $(\sqrt{15})^2=15, (\sqrt{50})^2=50$ であるから、
 $15 < a^2 < 50$ $a^2=16, 25, 36, 49$
 よって、 $a=4, 5, 6, 7$
- 5 (2) $x=0.\dot{1}4$ とおく。

$$\begin{array}{r} 100x = 14.14141414 \\ -) \quad x = 0.14141414 \\ \hline 99x = 14 \end{array}$$
 よって、 $x=\frac{14}{99}$
- $3.\dot{1}4=3+0.\dot{1}4=3+\frac{14}{99}=\frac{311}{99}$
- 6 (1) $(3+i)(5-6i)=15-18i+5i-6i^2$
 $=15-18i+5i+6=21-13i$
- (2) $\frac{5-10i}{4+3i}=\frac{(5-10i)(4-3i)}{(4+3i)(4-3i)}$
 $=\frac{20-15i-40i+30i^2}{16-9i^2}$
 $=\frac{-10-55i}{25}=-\frac{2+11i}{5}$
- (3) $(1-2i)^2=1-4i+4i^2$
 $=1-4i-4=-3-4i$
- 8 (1) $P=|-1-1|-3|3-(-1)|=|-2|-3|4|$
 $=2-12=-10$
- (2) $P=|\pi-1|-3|3-\pi|$
 $=\pi-1-3\{-(3-\pi)\}=\pi-1+3(3-\pi)$
 $=-2\pi+8$

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P.124-125

平方根と複素数のまとめ②

- 1 (1) $\sqrt{4}, \sqrt{\frac{9}{25}}, \sqrt{0.81}, \sqrt{1.21}, \sqrt{(-5)^2}$
 (2) $\sqrt{3}, -\sqrt{5}, \sqrt{(1-\sqrt{2})^2}$
- 2 (1) $\frac{5\sqrt{6}}{4}$ (2) $2\sqrt{2}-\sqrt{5}$
- 3 (1) $2\sqrt{3}$ (2) $4+2\sqrt{14}$
 (3) 4 (4) $18+12\sqrt{2}$
 (5) 1 (6) $-4\sqrt{15}$
 (7) $\frac{4\sqrt{15}}{15}$ (8) $-\frac{15\sqrt{2}}{2}$
 (9) $3+\sqrt{22}+3\sqrt{2}+\sqrt{7}$
 (10) $-\sqrt{30}+5\sqrt{3}+(-4\sqrt{3}+2\sqrt{30})i$
- 4 (1) $2\sqrt{3}$ (2) $18\sqrt{3}$
- 5 (1) $12\sqrt{11}$ (2) 40

解き方

- 2 (2) $\frac{3}{\sqrt{5}+\sqrt{8}}=\frac{3}{\sqrt{8}+\sqrt{5}}$
 $=\frac{3(\sqrt{8}-\sqrt{5})}{(\sqrt{8}+\sqrt{5})(\sqrt{8}-\sqrt{5})}$
 $=\frac{3(2\sqrt{2}-\sqrt{5})}{8-5}=2\sqrt{2}-\sqrt{5}$
- 3 (2) $(2\sqrt{7}-\sqrt{18})(2\sqrt{7}+\sqrt{32})$
 $= (2\sqrt{7}-3\sqrt{2})(2\sqrt{7}+4\sqrt{2})$
 $= (2\sqrt{7})^2+8\sqrt{14}-6\sqrt{14}-12(\sqrt{2})^2$
 $= 28+2\sqrt{14}-24=4+2\sqrt{14}$
- (5) $(\sqrt{3}-\sqrt{2})^2(\sqrt{3}+\sqrt{2})^2$
 $= \{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})\}^2$
 $= \{(\sqrt{3})^2-(\sqrt{2})^2\}^2=(3-2)^2=1$
- (7) $\frac{5\sqrt{3}}{3\sqrt{5}}-\frac{\sqrt{5}}{5\sqrt{3}}$
 $=\frac{5\sqrt{3}\times\sqrt{5}}{3\sqrt{5}\times\sqrt{5}}-\frac{\sqrt{5}\times\sqrt{3}}{5\sqrt{3}\times\sqrt{3}}$
 $=\frac{5\sqrt{15}}{15}-\frac{\sqrt{15}}{15}=\frac{4\sqrt{15}}{15}$
- 4 (1) $xy^2+x^2y=xy(x+y)$
 $= \{(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2})\}$
 $\quad \times \{(\sqrt{3}+\sqrt{2})+(\sqrt{3}-\sqrt{2})\}$
 $= (3-2)\cdot 2\sqrt{3}=2\sqrt{3}$
- (2) $x^3+y^3=(x+y)^3-3xy(x+y)$
 $= (2\sqrt{3})^3-3\cdot 1\cdot 2\sqrt{3}$
 $= 24\sqrt{3}-6\sqrt{3}=18\sqrt{3}$

$$\begin{aligned} \text{5 } a &= \frac{2}{\sqrt{11}-3} = \frac{2(\sqrt{11}+3)}{(\sqrt{11}-3)(\sqrt{11}+3)} \\ &= \sqrt{11}+3 \end{aligned}$$

$$\begin{aligned} b &= \frac{2}{\sqrt{11}+3} = \frac{2(\sqrt{11}-3)}{(\sqrt{11}+3)(\sqrt{11}-3)} \\ &= \sqrt{11}-3 \end{aligned}$$

$$\begin{aligned} \text{(1) } a^2 - b^2 &= (a+b)(a-b) \\ &= (\sqrt{11}+3+\sqrt{11}-3) \\ &\quad \times \{\sqrt{11}+3-(\sqrt{11}-3)\} \\ &= 2\sqrt{11} \cdot 6 = 12\sqrt{11} \end{aligned}$$

$$\begin{aligned} \text{(2) } a^2 + b^2 &= (a+b)^2 - 2ab \\ &= (2\sqrt{11})^2 - 2(\sqrt{11}+3)(\sqrt{11}-3) \\ &= 44 - 2(11-9) \\ &= 40 \end{aligned}$$